THE UNIVERSITY OF ALBERTA

RELEASE FORM

NAME OF OF AUTHOR: Robert Craig Keigo Kunimoto

TITLE OF THESIS: A Nonhomothetic Joint Multiple Output Multiple Input Translog Cost Function: An Estimation of the Technology of Canadian Agriculture

DEGREE FOR WHICH THESIS WAS PRESENTED: Master of Arts

YEAR THIS DEGREE GRANTED: 1983

THE UNIVERSITY OF ALBERTA

A NONHOMOTHETIC JOINT MULTIPLE OUTPUT MULTIPLE INPUT TRANSLOG COST FUNCTION: AN ESTIMATION OF THE TECHNOLOGY OF CANADIAN AGRICULTURE

by

© ROBERT CRAIG KEIGO KUNIMOTO

A THESIS
SUBMITTED TO THE FACULTY OF GRADUATE STUDIES AND RESEARCH
IN PARTIAL FULFILMENT OF THE REQUIREMENTS FOR THE DEGREE

MASTER OF ARTS

DEPARTMENT OF ECONOMICS

EDMONTON, ALBERTA
(FALL, 1983)

THE UNIVERSITY OF ALBERTA

FACULTY OF GRADUATE STUDIES AND RESEARCH

The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies and Research, for acceptance, a thesis entitled A Nonhomothetic Joint Multiple Output Multiple Input Translog Cost Function: An Estimation of the Technology of Canadian Agriculture submitted by Robert Craig Keigo Kunimoto in partial fulfilment of the requirements for the degree of Master of Arts.

DEDICATION

This work is dedicated to my father and mother who have taught me the most important lessons in life, to Harley and Chester whose questions aroused my curiosity, and to Susan whose friendship and understanding lessened the burden of this manuscript.

ABSTRACT

This thesis empirically examines the characteristics of the Canadian agriculture production technology disaggregating output into two distinct products. A multiple input multiple output joint nonhomothetic translog cost function with n - 1 cost share and m revenue share equations is estimated using pooled cross sectional time series data for the time period 1961 to 1979. In addition, a interregional comparison between Eastern and Western Canadian agriculture is conducted.

Statistical analysis focuses on global tests for alternative production structures such as homotheticity, homogeneity, log linearity and linear homogeneity. Four alternative specifications of global constant returns to scale were rejected. Furthermore, the rejection of global homotheticity suggests to view previous analysis employing the single output specification with caution.

Calculations are conducted to estimate the values of various summary measures such as the partial elasticity of substitution, own and cross price elasticities of factor demands, output elasticities of factor demands, marginal rates of product transformation, marginal cost of outputs and local overall returns to scale for each region and the Canadian aggregrate.

Local tests conducted on the agriculture technology reveals, at the point of approximation, agriculture is a increasing cost industry. Furthermore, local tests leads one to reject constant returns to scale at the point of

approximation for the two regions and the Canadian aggregate.

The empirical findings indicate there exists interregional differences in the agriculture technology utilized in Eastern and Western Canada. Morever, there exists significant intraregional differences in the technologies employed in the production of field crops and livestock/animal products. In conclusion, the agriculture technology of Eastern Canada, Western Canada and the Canadian aggregate is a nonhomothetic multiple output multiple input one.

ACKNOWLEDGEMENTS

I am grateful to my thesis committee: Professors M. L. McMillan, T. S. Veeman, W. E. Phillips and in particular Professor L. S. Wilson, my supervisor, who offered me constant encouragement throughout my M.A. program. In addition, I gratefully acknowledge Professor D. Gillen whose suggestions and comments directed me to this area of research.

Financial assistance from the Department of Economics at the University of Alberta was greatly appreciated. Moreover, the computer time made available to me by the Department of Economics and Political Science at the University of Saskatchewan is gratefully acknowledged.

My sincere thanks to Agriculture Canada and in particular Drs. Tung, Lopez and Nararayan for providing me with data. I am also indebted to Professors W. Bishopp, A. Buse, P. Dooley, L. St. Louis, R. Olley and J. Rude who through conversation and the provision of excellent answers to sometimes confused questions helped me avoid losing the direction of this research.

Finally, Susan deserves credit for enduring much hardship during the writing of thesis.

TABLE OF CONTENTS

CHAPTER PAGE

I. INTRODUCTION 1

1.1 Background 1

1.2 Literature Review 4

1.3 The Problem 11

1.4 Objectives 16

1.5 Outline 17

II LITERATURE REVIEW AND METHODOLOGY 19

2.1 Duality Theory 20

2.2 The Structure of the Production Technology . . 27

2.3 Summary Measures and Comparative Statics in General 31

2.4 Economies of Scale and Economies of Scope . . 37

2.5 Functional Specification 46

2.6 Flexible Functional Forms 51

III THE MODEL 58

3.1 The Multiple Input, Multiple Output Translog Cost Function 59

3.2 Restrictions for Alternative Structure 63

3.3 Functional Forms of the Summary Measures . . . 70

3.4 Economies of Scale and Economies of Scope . . 73

IV ECONOMETRIC ESTIMATION 78

4.1 Data 79

4.2 Estimation Methodology 83

4.3 Testing the Structure of Technology 89

4.4 Estimation Results and Summary Statistics . . 94

4.5 Economies of Scale and Economies of Scope . . 114

V SUMMARY AND CONCLUSIONS 120

5.1 Summary of Findings 120

5.2 Areas of Future Research 123

BIBLIOGRAPHY 127

APPENDIX 135

LIST OF TABLES

Table	Description	Page
1	Factor Input Values for Agriculture	14
2	Factor Input Ratios for Agriculture	15
3	Definition of Variables	84
4	Tests of the Production Structure	91
5	Tests of the Production Structure continued	92
6	Parameter Estimates for the Translog Cost Function	96
7	Goodness of Fit Statistics	97
8	Interpretation of Regional Dummy Variables	99
9	Estimated Elasticities of Substitution for Canadian Agriculture	101
10	Estimated Elasticities of Substitution for Western Canada	102
11	Estimated Elasticities of Substitution for Eastern Canada	103
12	Estimated Price Elasticities for Canadian Agriculture	105
13	Estimated Price Elasticities for Western Canada	106
14	Estimated Price Elasticities for Eastern Canada	107
15	Classification of Factor Inputs for Agriculture	109
16	Output Elasticities of Factor Demand	110
17	Classification of Output Elasticities of Factor Demand	113
18	Marginal Costs and Rates of Transformation	115
19	Local Tests of the Production Structure	117

LIST OF FIGURES

Figure	Description	Page
1	Multiple Output Cost Surface and Scale	39
2	Multiple Output Cost Surface and Scope	42
3	Structure of Tests for Homotheticity and Homogeneity	68
4	Restrictions on the Translog Cost Function for Homotheticity and Homogeneity	69

CHAPTER I: INTRODUCTION

1.1 Background

Agriculture is an important major industry in Canada accounting for 9.3 % of Canada's export trade.[1] The agriculture industry including the processing, wholesale and retail sector represents approximately 25 % of Canada's economic activity.[2]

The national food strategy conference of 1978 in its summary statement stated one of its eight objectives was to increase production and marketing efficiency.[3] A necessary condition to achieve this objective is an adequate knowledge of the technological structure of Canadian agriculture.

Historically, Canadian agriculture, on the supply side, has been characterized by technological change, net inflows of capital, net outflows of labor, increases in the capital-labor ratios and above average increases in

---

[1] Canada Yearbook 1980 - 81.

[2] Ibid.

[3] Ibid.

productivity. More recently, there has been a decline in the rate of growth in Canadian agriculture productivity.[4] As a result, several studies have empirically investigated the characteristics of an aggregate Canadian agriculture production technology utilizing time-series data.

In general, these studies have focused on estimating the growth in total factor productivity, the degree of technological change, returns to scale, and/or various summary statistics such as own price elasticity of factor demands, cross price elasticity of factor demands and elasticity of factor substitution. Moreover, most of these recent studies have employed relatively recent advances in three areas of economics: (1) duality, where a dual cost or profit function is used instead of the primal production function,[5] (2) the refinement of conditions of aggregation and the related theory of index numbers,[6] and (3) flexible functional forms and approximating

---

[4]The most recent study of the slow down in the 1960's and 1970's is Brinkman and Prentice (1983).

[5]For survey articles on duality see Diewert (1974b, 1978b, 1982), McFadden (1978), and Nadiri (1982). For an indepth presentation see Blackorby, Primont and Russell (1978), Fuss and McFadden (1978) and Varian (1978).

[6]For recent articles on aggregation and index numbers see Blackorby, Primont and Russell (1978) and Diewert (1976, 1978a).

functions.[7] These recent studies have contributed significantly to our statistical understanding of the Canadian agriculture production technology. In addition, these studies have provided us with an extremely useful guide showing the methodology of empirically implementing these relatively new advancements and techniques in economics.

The most recent advancement in the production analysis literature is an extension of the single output, multiple input costing approach to the multiple output, multiple input case.[8] All of the Canadian agriculture studies have utilized a single aggregate output approach.[9] This thesis is a first attempt to analyze Canadian agriculture using a multiple output, multiple input cost

---

[7]The use of flexible functional forms in econometric estimation has been rapidly increasing. For a selective literature review on flexible function forms see Appelbaum (1979), Berndt and Khaled (1979), Blackorby, Primont and Russell (1977, 1978), Burgess (1975), Denny (1974), Fuss, McFadden and Mundlak (1978), Lau (1974) and Wales (1977).

[8]The multiple output multiple input case is synonymous with the multi-output, multi-input case and multiproduct case.

[9]The author is aware of only one study using the multi-product, multi-input approach to agriculture. Ray (1982) analyzes U.S.A. agriculture.

approach explicitly testing the production structure.[10]

The following selective literature review provides a brief compendium of recent studies which investigated the characteristics of Canadian agriculture. The literature review is necessary in order to provide some background information on the production structure of Canadian agriculture.

1.2 Literature Review

Islam and Veeman (1980), employing a model comprised of five input share equations derived explicitly from a translog cost function augmented to incorporate technological change, empirically estimated the summary statistics of the Canadian agriculture production technology for the period 1961 to 1978. Although it is an excellent study, there exists one severe conceptual problem with their analysis which questions the empirical validity of their results. Islam and Veeman impose constant returns to scale on the production technology they seek to analyze.

---

[10] I have been recently informed of Islam (1982) who estimated several alternative specifications of Canadian agriculture. However, Islam (1982) does not empirically discriminate between alternative specifications and as a result, one does not know which one of the alternative models are empirically valid.

The maintained hypothesis of constant returns to scale may lead to biases in the empirical results if the production technology is in fact not homogeneous of degree one. Lopez (1980) has shown the assumption of constant returns to scale imposed on an empirically valid nonhomogeneous, nonhomothetic cost function can lead to statistically biased estimates of technological change. Furthermore, Brown, Caves and Christensen (1979) have shown the imposition of homogeneity and homotheticity, which are necessary conditions for constant returns to scale, on a nonhomothetic function leads to incorrect parameter estimates. Lau (1974) states[11],

> "The constant returns assumption may be dropped at virtually no cost. In many applications, especially in the empirical analysis of microeconomic units, it may not be appropriate to maintain the hypothesis of constant returns to scale."

Islam and Veeman by maintaining the constant returns to scale translog cost function are assuming homogeneity of degree one on the unknown true production technology. The use of this assumption is quite typical of earlier empirical costing studies and can be attributed to either

---

[11]Lau (1974) p. 180.

a misinterpretation of Diewert (1974b)[12] or the lack of a sufficiently large data set to estimate all the parameters involved in the nonhomogeneous case. It appears Islam and Veeman (1980) assume linear homogeneity because of the latter.[13] If the analyst has a large enough data set, then the hypothesis of constant returns to scale should be empirically tested rather than maintained.

Alternatively, Lopez (1980) using time series data for 1964 to 1977 estimated the summmary statistics with the aid of a modified Generalized Leontief cost function. Moreover, Lopez tested the hypotheses of fixed proportion production, constant returns to scale and homotheticity and rejected all three for Canadian agriculture. Therefore, according to Lopez´s results, the Canadian agriculture technology is nonhomothetic and the maintained hypothesis of constant returns to scale would lead to statistical biases.

Lopez (1980) maintained the hypothesis of the

---

[12]Diewert (1974b) made the theory of duality accessible to a wider audience with his classic article. However in his development of duality theory, Diewert assumes constant returns to scale on the production function for expositional simplicity. Consequently, Lau (1974) and Shephard (1974) correctly point out constant returns to scale is neither necessary nor sufficient for the duality results to hold.

[13]It appears Islam and Veeman (1980) had 18 observations and estimated 22 parameters directly.

modified Generalized Leontief cost function. Although there is no, a priori, theoretical grounds for choosing a particular flexible functional form, it may be the case the empirical results may be sensitive to the particular functional specification. If a generalized Box-Cox,[14] hybrid translog,[15] or a translog function[16] was used instead of the generalized Leontief,[17] then would the subsequent empirical results be consistent with Lopez? Furthermore, Lopez in his analysis used aggregate agricultural output. It is of interest to determine whether the statistical summary measures of Canadian agriculture are sensitive to the degree of aggregation.

Chan and Mountain (1981) engaged a translog primal production function to investigate the growth in total factor productivity in Canadian agriculture from 1953 to 1976 and attributed portions of growth to the rate of technical progress and returns to scale. The hypothesis of linear homogeneity of the production technology is

---

[14]See Applebaum (1979), Berndt and Khaled (1979), or Kiss, Karabadjian and Lefebvre (1981).

[15]See Caves, Christensen and Tretheway (1980), Fuss and Waverman (1978), or Kiss, Karabadjian and Lefebvre (1981).

[16]The transcendental logarithmic function or translog for short was introduced by Christensen, Jorgenson, and Lau (1973).

[17]The generalized Leontief function was orginally formulated by Diewert (1971).

rejected at both the 10% and 5% level of significance.[18]

Estimating a restricted profit function, Danielson (1975) analyzed Canadian agriculture for the period of 1946 to 1970 and rejected the existence of an aggregate profit function. Danielson's primary contribution was to examine and show the problems associated with data collection and data refinement. The rejection of the existence of an aggregate profit function may be as a result of his data set or his choice of a functional form rather than the nonexistence of such a function.

Islam (1982) estimated the agriculture production technology for Canada and Western Canada for the period of 1961 to 1978 employing several alternative sets of restrictions. Parameter estimates were arrived at by specifying four variants of the translog cost function. The homothetic joint output model with technological change,[19] the homothetic single output model with technological change, and the nonhomothetic single output model with and without technological change are all estimated using a translog cost function. However, Islam (1982) does not empirically test the validity of the

---

[18]Chan and Mountain (1981) p.12.

[19]Islam (1982) uses the expression "joint output" to mean "multiple output". The multiple output literature notes the distinction between the two expressions.

restrictions he has imposed on his model and therefore does not allow him to make any statistical conclusions between the alternative variants of his model. In addition, Islam (1982) does not test for methods of aggregating output from the multiple output case to the single output case.

Lopez and Tung (1982), using pooled Canadian data and a refined data base, building upon Lopez (1980), investigated profit maximizing behavior and the implications of energy conservation measures on Canadian agriculture. It appears Lopez and Tung (1982) utilized pooled time series and cross sectional data for the period 1961 to 1979 to increase the size of their data set. The authors do explore to some extent the degree of the possible interregional differences between the agriculture production technology of Eastern and Western Canada.[20] However, Lopez and Tung (1982) employed the generalized Leontief function and a single aggregate output as was the case in Lopez (1980).

Finally, Lopez (1982) utilizing a variable profit function examined the long run responses of Canadian

---

[20]Lopez and Tung (1982) used dummy variables in their first order equations to capture interregional variations and concluded Western Canada appears to have a more land, labor and other intermediate input intensive technology compared to Eastern Canada.

agriculture to higher Canadian energy prices. The analysis is based upon the summary statistics found in Lopez (1980).

There are several points of interest which are common to all of the above cited studies. First, all of the aforementioned studies employed highly aggregated time-series national data to estimate the production technology of Canadian agriculture.[21] In spite of an obvious need for pursuing a more disaggregated approach, relatively little effort has been devoted to this area. It is of interest to examine the agriculture production technology at either the provincial level or the regional level to determine the extent of interprovincial or interregional variations.

Second, all of the above mentioned studies relied on either the generalized Leontief or translog functional specification. Lopez, in all of his studies, utilized the generalized Leontief form. It is of interest to determine to what extent his empirical results are sensitive to that

---

[21]One exception is Lopez and Tung (1982) who use pooled data comprised of Eastern and Western Canadian data.

functional specification.[22]

Finally, all of the reviewed studies maintain the existence of a single aggregate output. This is equivalent to assuming separability of input prices and outputs in the cost function.[23] It may be the case the use of a single aggregate output leads to statistical biases in the estimated summary measure if the production technology is a multiple output one and the method of output aggregation is ad hoc.

1.3 The Problem

Although empirical investigation of a particular economic phenomena using aggregate data may yield useful results, it may conceal empirically valid observations available from the utilization of a more disaggregated approach. Public policy based on results derived from aggregate analysis may lead to inappropriate recommendations. Consequently, it is of interest to

---

[22]Islam and Veeman (1980) used the translog cost function while Chan and Mountain (1981) employed the translog production function. The problem with the former is the imposition of constant returns to scale and a potential problem with the latter is the measurement problems associated with estimation from the primal side.

[23]The conditions for separability of input prices from outputs will be discussed in a later chapter.

investigate the characteristics of Canadian agriculture disaggregated in two ways. First, it is useful to examine the agricultural output and treat it as two distinct product types: field crops and livestock/animal products. Second, due to the heterogeneity of Canadian agriculture, it is of interest to investigate the interregional differences in the agriculture technology.[24]

It is of interest to examine Canadian agriculture disaggregating output into a multiproduct cost function to determine to what extent the summary measures reported in the above cited studies are sensitive to the degree of output aggregation. In addition, by using a multiproduct approach one can test for cost complementarities, economies of scope, and calculate marginal rates of product transformation which are unavailable in the single output case.[25]

The heterogeneity of Canadian agriculture is reflected in part by the interregional differences in the agriculture output mix, factor endowments, factor usage,

---

[24]Originally, I started to examine the interprovincial differences in Canadian agriculture. However, due to the unavailability of a data set, I quickly abandoned the approach and moved to a more tractable perspective.

[25]Cost complementarities, economies of scope and marginal rates of product transformation are defined in a subsequent chapter.

output specialization and factor proportions. Therefore, as a brief introduction to the problem, an examination of interregional variations in factor usage and factor proportions is presented.

Table 1 represents the levels of the factors of production employed in agriculture by province in the census year 1976. Generally and discursively, the table indicates the prairie provinces exhibited the highest absolute factor usage of land, labor and capital by region and this usage declines as one moves east or west.

Alternately, table 2 specifies the factor input ratios observed in agriculture for the same year. The cross sectional observation indicates the value of capital per unit of labor and the land-labor ratio was the largest in the prairie provinces and generally declines moving either east or west.

The discussion above does clearly indicate there exists differences in factor usage and factor proportions among provinces for the selected year 1976. How much of these interprovincial variations can be, in general, attributed to differences in provincial technologies and specifically to differences in the rate of technological progress, scale effects and output composition? Data limitations require an interregional comparison between Eastern and Western Canada rather than an interprovincial

Table 1: Factor Input Values for Agriculture

Province	Land[a]	Capital[b]	Labor[c]
Atlantic	1415	240	21
Quebec	5923	863	74
Ontario	11069	1940	113
Prairies	88967	5645	247
British Columbia	1911	346	18

a. Improved land in use in 1976
(thousands of acres) Source: Census of Canada

b. Value of machinery and equipment in 1976
(millions of dollars) Source: Statistics Canada

c. Paid and unpaid workers in agriculture in 1976
(thousands of workers) Source: Statistics Canada

Table 2: Factor Input Ratios for Agriculture[a]

Region	Land/Labor	Capital/Labor
Maratimes	67.38	11.43
Quebec	80.04	11.66
Ontario	97.96	17.17
Prairies	360.19	22.85
British Columbia	106.17	19.22

a. Calculated by dividing the 1976 factor input values by labor as reported in Table 1.

comparison.

The purpose of this thesis is to investigate, analyze and examine the characteristics of the multiple output agricultural production technologies adopted by these two regions in Canada in anticipation of inferring further interregional diffences.

1.4 Objectives

The problem, as defined above, suggests the development and implementation of a comprehensive and general quantitative model for use in empirically analyzing interregional differences in the production technology of Canadian agriculture. As a result, the objectives of this thesis are as follows:

1. to review the literature with respect to duality in production and functional forms,
2. to specify a model and methodology employing a multiple output, multiple input cost function which can be implemented to determine the degree of interregional variations in Canadian agriculture,
3. to examine the summary statistics of Canadian agriculture derived from the multiple output, multiple input cost model,
4. to determine empirically if there exists interregional differences in agricultural production technologies,
5. to estimate the degree and extent of the interregional differences and attribute them to the appropriate explanatory factors.

In short, the objective of this thesis is to provide an economic investigation of the multiple input, multiple output production technologies representative of Canadian agriculture at the regional level.

1.5 Outline

The organization of this thesis is as follows. A brief literature survey on duality and functional forms in the multiple input, multiple output case is provided in Chapter 2. Moreover, the general forms of the various summary measures are presented. In addition, the notions of homogeneity, homotheticity, linear homogeneity, economies of scope and cost complementarities in the multiproduct case are discussed.

In Chapter 3, the regional multiple input, multiple output agriculture cost functions are specified. The nonhomothetic, joint, multiple input, multiple output translog cost function is introduced. Moreover, the exact functional forms of summary statistics to be estimated are developed in this chapter in conjunction with the exact forms of the hypotheses to be tested.

Chapter 4 presents the econometric aspects of

estimation and tests the hypothesis of a general global structure. Subsequently, a maintained hypothesis of a particular global structure is imposed on the production technology for tractability such that an interregional comparison can be made. The estimated summary statistics and the empirical results of various local test are also provided.

Chapter 5 provides a summary of the major findings of this thesis. Moreover, the limitations of this study are discussed. The conclusions of this study and directions for subsequent analysis are also provided in this chapter.

CHAPTER II: LITERATURE REVIEW and METHODOLOGY

This chapter is comprised of a selective literature review of production theory. Section 2.1 examines the use of duality theory to estimate the characteristics of a multiple input, multiple output production technology. The sufficient conditions required of the dual cost function such that it succinctly describes the production technology are presented. In addition, the advantages associated with use of the dual approach are highlighted.

In section 2.2, restrictions on the general production structure are discussed. In particular, the notions of homotheticity and homogeneity of the production structure and their subsequent effects on the cost function are discussed.

In section 2.3, the general forms of various summary measures such as the Allen-Uzawa partial elasticities of substitution, the partial cross price elasticities of factor demands, the own price elasticities of factor demands, the output elasticity of factor demands, and the marginal rate of product transformation are presented.

A discussion of product-specific economies of scale and overall economies of scale is presented in section 2.4. Furthermore, a measure for the existence of economies

of scope is presented in the context of a multiproduct industry.

Section 2.5 discusses the functional specifications of the production technology in general. The characteristics of flexibility are discussed in the context of flexible functional (approximating) forms and generalized (exact) forms in section 2.6.

2.1 Duality Theory

The purpose of this section is to provide a brief description of the duality between production and cost functions in order to make this thesis self contained. Moreover, the pragmatic and theoretical advantages associated with the dual approach vis-a-vis the primal approach are highlighted.[26]

The fundamental principle of duality in production states if there exists cost minimization and input price taking behavior then the cost function of a firm summarizes all of the economically relevant aspects of its

---

[26]The presentation is given without proofs since they are easily found elsewhere. A more detailed description of the dual approach to micro theory including proofs can be found in Shepard (1970), Uzawa (1964), McFadden (1978), Diewert (1971, 1974b, 1978b, 1982), Fuss and McFadden (1978), Blackorby, Primont and Russell (1978), Nadiri (1982), and Varian (1978).

technology.[27] Consequently, under the appropriate regularity conditions, the production technology utilized by a firm or an industry can be alternately and equivalently described and analyzed by either the primal function (production function or the transformation function) or the dual function (cost function or profit function).[28] As a result, the analyst can specify a well behaved dual cost function without explicitly deducing it from the primal function and be assured it characterizes the economic information of production if and only if the cost function satisfies certain regularity conditions.

A set of sufficient conditions required of a cost function such that there exists an unique relationship between the primal and dual functions is as follows. Let the cost function be represented by a twice differentiable continuous function such as,

$$C = C(P,Q) \tag{1}$$

where:

C represents total costs,

P is a vector of input prices P_1, P_2,..., P_n., and

---

[27] Varian (1978), p.38.

[28] There also exists a duality between the derivatives of the primal and dual functions.

Q is a vector of outputs $Q_1, Q_2, \ldots, Q_m$.

If the multiple input, multiple output cost function defined in (1) is:[29]

1. positively linear homogeneous in P such that
   $$C(tP,Q) = tC(P,Q)$$
   for all $t \geq 0$, $Q > 0$, $P > 0$
2. strictly positive for all positive input prices and every positive value of Q such that
   $$C(P,Q) > 0$$
   for all $P > 0$, $Q > 0$
3. nondecreasing in input prices such that
   $$C(P',Q) \geq C(P,Q)$$
   for all $P' \geq P$
4. concave in input prices such that
   $$C(tP + (1-t)P',Q) \geq$$
   $$tC(P,Q) + (1 - t)C(P',Q)$$
   for $0 \leq t \leq 1$

then via the duality mapping the cost function (1) represents a well behaved production transformation function such as,

$$F(Q,X) = 0 \tag{2}$$

where:

---

[29]See Varian (1978), pp. 39-43 or Fuss and Waverman (1978), p. 7.

Q is a vector of outputs $Q_1, Q_2, \ldots, Q_m$.

X is a vector of inputs $X_1, X_2, \ldots, X_n$.

As a result, one needs only to specify a well behaved cost function meeting the conditions 1 to 4 and be assured it represents the economic relevant information of a production technology.[30]

The advantages associated with the use of the cost function as opposed to the transformation or production function are:

1. the cost function utilizes economic observables such as input prices where as the production function employs variables such as the quantities of inputs which may be difficult to quantify.[31]

2. empirical analysis using the cost function is computationally simpler and allows one to test a wider range of hypotheses.[32]

3. factor demands and/or cost share equations are derived easily by the use of Shephard´s lemma.

---

[30]Typically, the sufficient conditions on the cost function for the existence of the duality mapping are shown in the case of a single output cost and production function. For an elaboration of the multiple input, multiple input case see Lau (1980), Nadiri (1982), Denny, Fuss and Waverman (1981), Fuss (1981), Fuss and Waverman (1978), Panzer and Willig (1981), McFadden (1978), Hall (1973) and Baumol, Panzer and Willig (1982).

[31]See Varian (1978) or Intrilligator (1978) for problems associated with capital measurement.

[32]Nadiri (1982) p.449.

4. summary statistics such as the various elasticity measures are easily calculated in terms of the cost function and its derivatives.

5. the use of the cost function expands the set of possible functional forms describing the production technology when the function is not a self dual.[33]

6. one can specify a well behaved cost function under the appropriate regularity conditions and be assured it represents some production technology.[34]

Once the cost function (1) has been specified such that it represents the economic information of (2), then the use of Shepard's lemma (Shepard, 1958) allows one to obtain the factor input derived demand functions,

$$X_i = \delta C(P,Q)/\delta P_i = X_i(P,Q) \qquad i = 1, 2, 3,..., n. \quad (3)$$

where:

X_i represents the quantity demanded of the i^{th} input

P_i represents the price per unit of the i^{th} input.

Alternatively, one can derive the factor input share equations,

---

[33]Lau (1974), p. 185.

[34]With particular functional forms of the transformation or production function such as the translog, it is impossible to derive the corresponding deduced cost function.

$$S_i = \delta \ln C / \delta \ln P_i = S_i(P,Q) \quad i = 1, 2, 3, \ldots, n. \quad (4)$$

where:

S_i represents the cost share of the i^{th} input such that $S_i = P_i \cdot X_i / C$.

Thus, rather than specifying a functional form for the production function and deriving the input demand functions therefrom , one can specify a cost function directly which satisfies conditions 1 to 4 and then apply Shephard´s lemma to obtain the input demands or the cost share equations.

Consequently, the use of the input demand functions or the cost share equations allows one to develop estimatable forms of certain summary measures which succinctly describes the production technology of an industry or firm. One develops and estimates certain statistics such as various elasticity measures conveniently in terms of the cost function and its partial derivatives.

Similarly, one can derive the marginal cost of producing an extra unit of any particular output by,

$$MC_y = \delta C(P,Q) / \delta Q_y \qquad y = 1, 2, 3, \ldots, m. \quad (5)$$

or the output elasticity of total cost for any particular

output by,

$$E_{cy} = \delta \ln C(P,Q) \,/\, \delta \ln Q_y \qquad y = 1, 2, 3, \ldots, m. \quad (6)$$

If the industry is characterized by perfect competition and the firms and industry are in equilibrium, then (6) becomes the revenue share of the y^{th} output,[35]

$$R_y = \delta \ln C(P,Q) / \delta \ln Q_y \qquad y = 1, 2, 3, \ldots, m. \quad (7)$$

The system (1), (3), and (5) or (1), (4), and (7) constitutes the basic general full model used to analyze Canadian agriculture. The exact specification of the model employed is dependent of the functional specification of the cost function (1). A discussion of functional specification and the exact model employed is postponed to a later section of this thesis.

Although economic theory provides little guidance in choosing an appropriate functional form of (1), theory does dictate that certain conditions be met such that the specification is consistent with production theory. Neoclassical production theory imposes several

---

[35]See appendix A for the proof of equation (7)

restrictions upon either of the two general models above. A well behaved neoclassical cost function must be homogeneous of degree one in input prices. Linear homogeneity of the cost function in P implies, by Euler's theorem, the following restrictions:[36]

a. the adding up condition where the sum of all costs equal total costs such that

$$\sum_i P_i \cdot X_i(P,Q) = C(P,Q)$$

b. Cournot's aggregation condition such that

$$\sum_i P_i \, \delta X_i(P,Q)/\delta P_j = 0$$

c. Engel's aggregation condition such that

$$\sum_i P_i \cdot C_{iy}(P,Q) = C_y(P,Q)$$

d. Furthermore, symmetry according to Young's theorem results in the additional restriction

$$C_{ij} = C_{ji}$$

These restrictions are then translated into parameter restrictions, the exact form determined by the functional specification of the cost function.

2.2 The Structure of the Production Technology

The cost function (1) is a general nonhomothetic joint cost function. Given a multiple input, multiple output cost function, it is of interest to examine whether the underlying production structure is globally homothetic

---

[36] Fuss, McFadden and Mundlak 1978, p. 232

or nonhomothetic. A technology is globally homothetic in output if the cost minimizing expansion path, given input prices, is a ray from the origin and the slopes of the isoquants on the expansion path are invariant to output (Silberberg, 1978). Denny and May (1978b) have identified two economic properties of the cost function which is a dual to a homothetic production technology:

> "First, the ratio of any two factor demand equations is independent of the output level. Second, the elasticity of total or average cost with respect to output is independent of factor prices."

The production technology is weakly homothetic if the first condition is met. A weakly homothetic production technology is sufficient for linear expansion paths (Denny and May, 1978b). Weak homotheticity of the production structure implies input prices in the dual cost function are weakly separable from output such that the cost function can be written as:[37]

$$C = G(P, H(Q)) \tag{8}$$

Moreover, the production technology is strongly

---

[37]See Hall (1973), p. 890 for this specification. However, Hall (1973) does not use the "weak" terminology.

homothetic if conditions one and two are satisfied. According to Denny and May (1978b), if the production structure is strongly homothetic then the dual cost function can be written in multiplicatively separable form as:

$$C = G(P) \cdot H(Q) \tag{9}$$

It is important to test the production structure for homotheticity and hence the cost function for input prices and output separability for two reasons. First, the test for separability of input prices and outputs in the multiple output cost case is the required test for the existence of a consistent output aggregator (Fuss and Waverman, 1978). Consequently, if the empirical results confirm separability of outputs from input prices then one can legitimately use the single aggregate output specification of the cost function. Second, global homotheticity is a necessary condition for global homogeneity. If the production technology is not homothetic, then the imposed assumption of homogeneity will result in a specification bias.

Given the production function is globally homothetic, a production function is homogeneous of degree r if when all inputs are multiplied by a scalar, t, then output increases by a factor of t^r. Subsequently for the dual

cost function, Brown, Caves and Christensen (1979) have shown that homogeneity of degree r of the production function implies homogeneity of degree 1/r of the cost function such that:

$$t^{1/r}C = G(P, H(t^{r}Y)) \tag{10}$$

for the weak homotheticity and

$$t^{1/r}C = G(P) \cdot H(t^{r}Y) \tag{11}$$

in the case of strong homotheticity.

Homogeneity of the production structure indicates equal spacing of isoquants along a ray from the origin. A homogeneous in output cost function is homogeneous of degree one in outputs if and only if r = 1 in the above specification. This implies as outputs expand by a given proportion, cost increase by the same proportion.

In several production studies, linear homogeneity of the production structure and hence of the dual cost function has been assumed. It is the procedure of this thesis to test the production structure to determine if in fact it is homothetic, homogeneous, or the more restricted linear homogeneous.

2.3 Summary Measures and Comparative Statics in General

Once the parameters of the model have been estimated, then one can calculate or estimate several summary statics which succinctly summarize the behavior of the industry. One can develop certain statistics such as various elasticity measures conveniently in terms of the cost function and its partial derivatives.

The Allen partial elasticity of substitution (AES) measures the effect on relative factor quantities resulting from a change in relative factor prices, holding output and other input prices constant and allowing all factor inputs to adjust. Uzawa (1962) has shown that the partial elasticity of substitution can be represented in terms of the dual cost function and its partial derivatives as:

$$AUES_{ij} = C\ C_{ij} \ /\ C_iC_j \quad i,j = 1, 2, 3, \ldots, n. \qquad (12)$$

where

C represents the total cost function,

C_i represents the first partial derivative of C with respect to the price of input X_i,

C_j represents the first partial derivative of C with respect to the price of input X_j,

C_{ij} represents the second partial derivative of C with respect prices of inputs X_i and X_j.

Uzawa (1962) proved the AES_{ij} derived from the primal side is equivalent to $AUES_{ij}$ derived from the dual cost function in the case where the production function is homogeneous of degree one. Binswanger (1974b) showed the two are equivalent regardless of the degree of homogeneity on the production function.

The sign of any Allen-Uzawa partial elasticity of factor substitution allows one to classify factor inputs i and j in the following manner:

1. If $AUES_{ij} > 0$ then inputs i and j are substitutes in production.
2. If $AUES_{ij} < 0$ then inputs i and j are complements in production.
3. If $AUES_{ij} = 0$ then inputs i and j are independent.

The partial cross price elasticities of factor demands by definition measures the responsiveness of the demand for one input as a result of a price change in another input holding output constant. The partial cross price elasticity of factor demands is conventionally defined as,

$$E_{ij} = (\delta X_i/\delta P_j)\cdot(P_j/X_i) \quad i, j = 1, 2, 3, \ldots, n. \qquad (13)$$

and measures the responsiveness of the demand for the i^{th} input to a change in the price of the j^{th} input.

In terms of the cost function and it's partial derivatives, the partial cross price elasticity of factor demands can be expressed as:

$$E_{ij} = (C_{ij}\ P_j) / C_i \qquad i, j = 1, 2, 3, \ldots, n. \quad (14)$$

It has been shown that E_{ij} and $AUES_{ij}$ are related by:[38]

$$E_{ij} = AUES_{ij} \cdot S_j \qquad i, j = 1, 2, 3, \ldots, n. \quad (15)$$

where:

$AUES_{ij}$ is defined in (12),

S_j is the cost share of the input j defined by (4).

Therefore, the analyst can estimate either E_{ij} or $AUES_{ij}$ and infer the other with the use of the appropriate share equation or vice-versa. It is clear from (15), the sign of $AUES_{ij}$ uniquely determines the sign of E_{ij} since $0 < S_i < 1$. As a result, one can classify any pair of factor inputs by the following:

1. $E_{ij} > 0$ <==> $AUES_{ij} > 0$ ==> i and j are substitutes,
2. $E_{ij} < 0$ <==> $AUES_{ij} < 0$ ==> i and j are complements,

---

[38]Allen (1938) originally showed (15) was true using AES.

3. $E_{ij} = 0 \iff AUES_{ij} = 0 \Longrightarrow$ i and j are independent,

Similarly, the partial own price elasticity for the i^{th} factor demand measures the responsiveness of the i^{th} factor demand to changes in its own price. Conventionally, the partial own price elasticity of factor demand is expressed as:

$$E_{ii} = (\delta X_i/\delta P_i) \cdot (P_i/X_i) \qquad i = 1, 2, 3, \ldots, n. \qquad (16)$$

Using the dual cost function and its partial derivatives, the partial own price elasticity of the i^{th} input can be alternatively expressed as:

$$E_{ii} = (C_{ii}\ P_i)/C_i \qquad i = 1, 2, 3, \ldots, n. \qquad (17)$$

The partial own price elasticity of factor demands will have a negative sign which indicates a downward sloping to the right input demand curve. The magnitude of the own price elasticity for any factor input allows one to classify the input into one of the following categories:

1. If $E_{ii} = 0$, then the demand for the i^{th} input is perfectly inelastic.
2. If $-1 < E_{ii} < 0$, then the demand for the i^{th} input is relatively inelastic.

3. If $E_{ii} = -1$, then the demand for the i^{th} input is unitary elastic.

4. If $E_{ii} < -1$, then the demand for the i^{th} input is relatively elastic.

5. If $E_{ii} \longrightarrow -\infty$, then the demand for the i^{th} input is infinitely elastic.

The Allen-Uzawa partial elasticity of substitution and the partial cross price and own price elasticities of factor demands evaluate economic phenomena holding output constant. A measure which explicitly considers the effect on factor demands as a result of a change in output is called the output elasticity of factor demand. The output elasticity of factor demand E_{iy} measures the responsiveness of factor demands to a change in output. In the multiple output case, the output elasticity of the i^{th} factor demand calculated from the primal side can be expressed as:

$$E_{iy} = (\delta X_i/\delta Q_y)\cdot(Q_y/X_i) \qquad \begin{array}{l} i = 1, 2, 3, \ldots, n. \\ y = 1, 2, 3, \ldots, m. \end{array} \qquad (18)$$

In terms of the dual cost function the output elasticity of the i^{th} factor demand can be expressed as:

$$E_{iy} = C_{iy}\cdot Q_y/C_i \qquad \begin{array}{l} i = 1, 2, 3, \ldots, n. \\ y = 1, 2, 3, \ldots, m. \end{array} \qquad (19)$$

where:

$$C_{iy} = \delta^2 C/\delta P_i \delta Q_y,$$
$$C_i = \delta C/\delta P_i$$

Since Q_y and C_i will be positive, the sign of the output elasticity of factor demand depends on the sign of C_{iy}. One can classify each factor of production according to the sign and magnitude of E_{iy} in one of the following ways:[39]

1. If $E_{iy} > 1$ then the i^{th} input is superior.
2. If $0 < E_{iy} < 1$ then the i^{th} input is normal.
3. If $E_{iy} < 0$ then the i^{th} input is inferior.[40]

A superior input by definition indicates as outputs expands proportionately input use of a particular factor of production increases by a greater proportion. Alternatively, an inferior input reflects a decrease in the proportion of use in the input given a proportional increase in output. Finally, a normal input suggests as output increases by a given proportion the use of the i^{th} input increases by less than proportionally.

---

[39]Gould and Ferguson (1980), p. 167 use this terminology but in reference to expenditure elasticity of factor demands.

[40]Lopez and Tung (1982) use E_{iy} and the terminology of superior, normal and inferior inputs.

In addition to the above elasticities measures, one can examine the production tradeoff in a multiple output technology as represented by a transformation curve. The marginal rate of transformation of output y for output r shows the number of units of y which must be given up in order to produce an extra unit of output r. Employing a cost function the marginal rate of tranformation of output y for output r can be expressed as:[41]

$$MRT_{yr} = (\delta C/\delta Q_r)/(\delta C/\delta Q_y) \tag{20}$$

2.4 Economies of Scale and Economies of Scope

Recent literature examining a multiproduct cost function have identified various effects on the cost function as a result of a change in output (s) or the product mix. Given a multiple input, multiple output cost function, one can test for product-specific economies of scale, overall economies of scale, and economies of

---

[41] See Darrough and Heineke (1978) and Denny and Pinto (1978) who estimated the marginal rate of product transformation.

scope.[42] This analysis can be readily adapted to examine the cost/output effects in an industry such as agriculture.

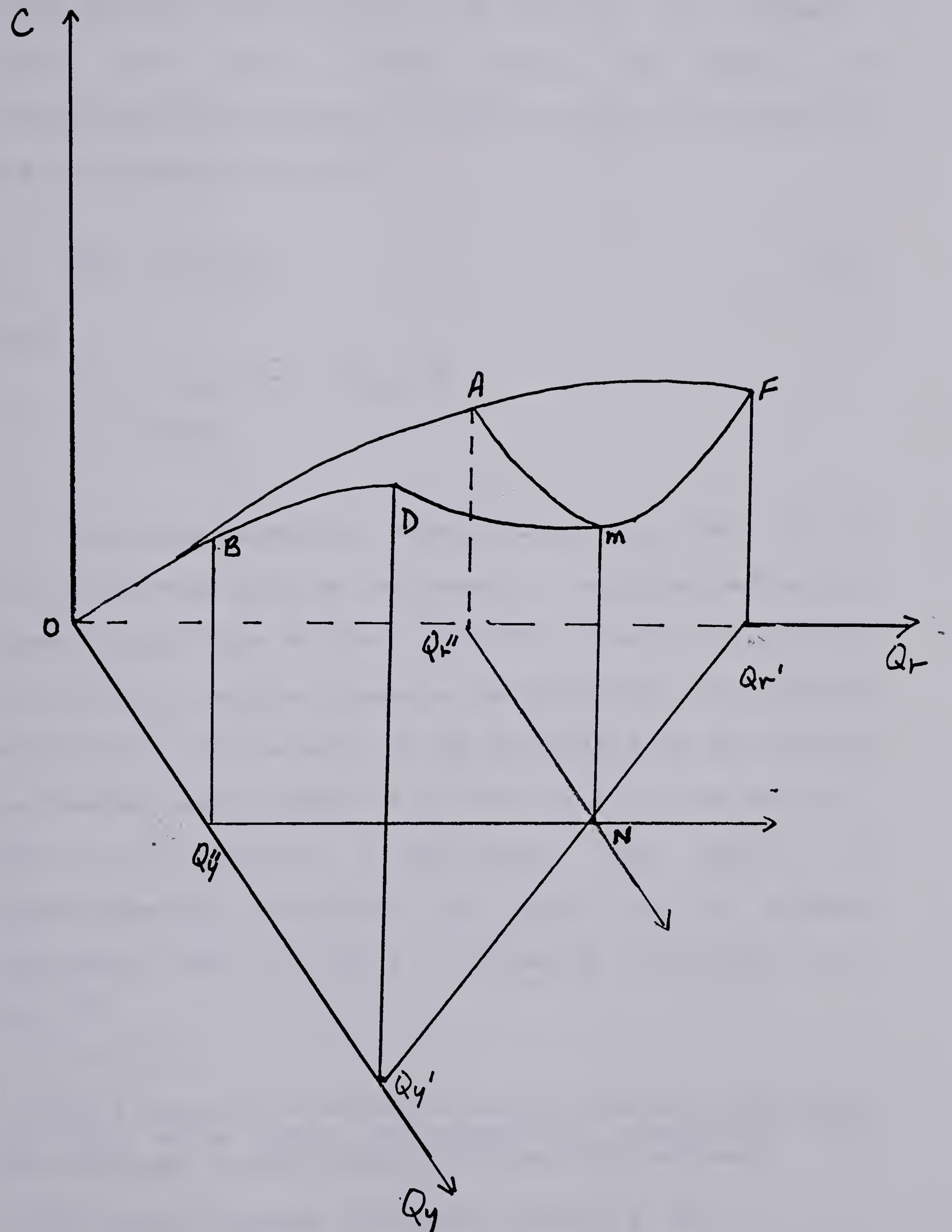
Given a multiproduct cost function, product-specific scale economies examine how costs change as the output of one product changes holding the level of other outputs fixed.[43] Suppose the multi-output cost function is given by the hypersurface as in Figure 1. The vertical axis C represents total costs. Axis labelled Q_y and Q_r represents output levels of product y and product r, respectively. It then follows, any point on the hypersurface ODF represents the total cost of producing a particular combination of Q_y and Q_r. As an example, the cost of producing Q_y'' and Q_r'' together is represented by the vertical distance NM. Product-specific scale economies for Q_y analyzes the responsiveness of costs along AM as Q_y is increased holding Q_r constant at Q_r''. Similarily, product-specific scale economies for Q_r measures the responsiveness of costs along BM as Q_r is increased holding Q_y constant at Q_y''.

---

[42]For an excellent survey see Bailey and Friedlander (1982). In addition, see Baumol, Panzer and Willig (1982), Fuss and Waverman (1978), Gillen and Oum (1983), Nadiri (1982), and Panzer and Willig (1977, 1981).

[43]Bailey and Friedlander (1982), p. 1030.

Figure 1



Global product-specific economies of scale for Q_y measures the responsiveness of cost as Q_y is increased along AM from A where $Q_y = 0$ to M where $Q_y = Q_y''$. Baumol, Panzer and Willig (1982) define the degree of product-specific economies of scale for the r^{th} product in the two product case as:[44]

$$SE_r = IC_r/Q_rC_r \tag{21}$$

where

$$IC_r = C(Q_y, Q_r) - C(Q_y, 0)$$
$$C_r = \delta C/\delta Q_r$$

According to Baumol, Panzer and Willig (1982) IC_r is the incremental cost of the product r which by definition shows the addition in total costs as a result of producing all products together compared to producing all products excluding r. Furthermore, IC_r/Q_r is defined as the average incremental cost of the r^{th} product and C_r is the marginal cost of producing r. Therefore, the degree of product-specific economies of scale is the average incremental cost of product r divided by the marginal cost of r.[45]

---

[44]The literature on scale and scope typically suppresses the input price vector for expostional simplicity. The notation used in this section follows this approach.

[45]See Baumol, Panzer and Willig (1982), p. 68.

A local measure of product-specific economies of scale can be represent by the output cost elasticity of the r^{th} product defined by:

$$\delta \ln C/\delta \ln Q_r = (Q_r/C) \cdot (\delta C/\delta Q_r) \quad (22)$$

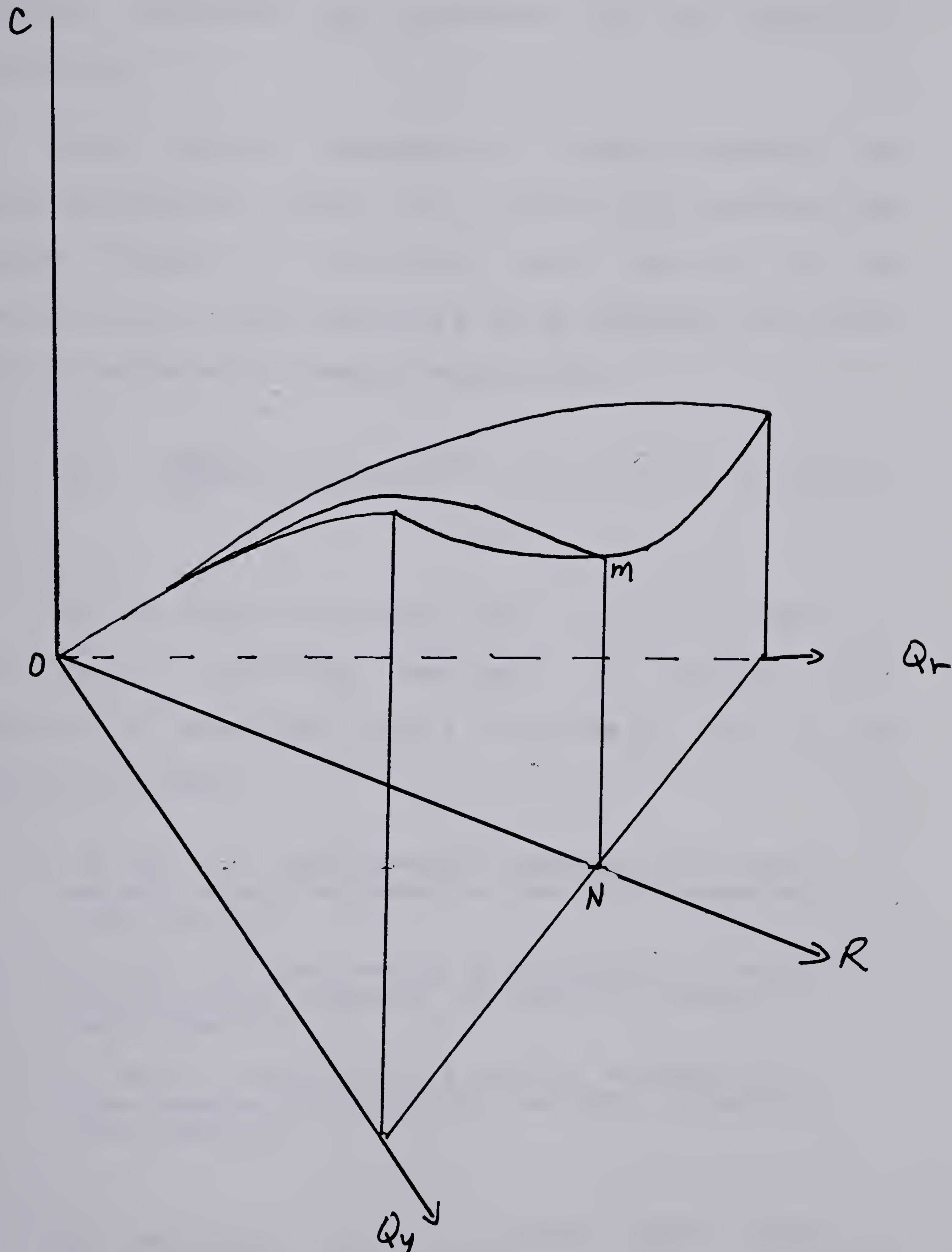
However, Fuss and Waverman (1978) have shown that the use of the local measure of product-specific economies may lead to a conflict with the overall returns to scale definition.

Overall scale economies measures the responsiveness of costs as a result of a change in all outputs. More specifically, ray overall economies of scale measures the responsiveness of costs as a result of expanding all outputs by an equal proportion along a ray from the origin such that the output mix is assumed to be constant (Bailey and Friedlaender, 1982).

In Figure 2, the expansion of outputs occurs along the ray OR such that the compositon of output Q_y/Q_r is constant. Overall scale economies measures the responsiveness of costs along OM as output is changed along OR.

A global test for overall economies of scale would examine the change in costs measured along OM as a result

Figure 2



of output expanding along the ray from O to R. The global test for overall economies of scale was discussed in the section discussing the structure of the production technology.

Local overall economies of scale examines the responsiveness of costs as a result of expanding the output bundle by relatively small amounts in the neighborhood of some arbitrary point assuming the output mix is unchanged and may be measured by:

$$ES = \left(\sum_{Y} \delta \ln C \,/\, \delta \ln Q_{y}\right)^{-1} \qquad y = 1, 2, 3, \ldots, m. \quad (23)$$

One can determine whether the agriculture industry is an overall increasing, decreasing or constant cost industry by examining overall economies of scale in the following manner:

1. If ES > 1, then overall economies of scale exists which reflects an overall decreasing cost industry.
2. If ES < 1, then overall diseconomies of scale exists which reflects an overall increasing cost industry.
3. If ES = 1, then there is neither economies nor diseconomies of scale which reflects a constant cost industry.

By definition, product-specific scale economies measures the responsiveness of costs as a result of

increasing one output while holding the level of all other outputs constant. This necessarily implies the measures reflects an nonoptimal change in the output mix such that the output ratio, Q_y/Q_r, is changing.

A new concept and measure has been developed to analyze the effects of a change in the output mix on total costs. Economies of scope compares the cost of producing a number of products jointly as opposed to the cost of producing them separately. In the two product case, there are economies of scope when its less costly to produce two products together than produce the two products separately (Panzer and Willig, 1981). In Figure 1, economies of scope exist if the cost of producing Q_y" and Q_r" jointly measured by the vertical distance NM is less than the cost of producing Q_y" measured by the vertical distance Q_y"B plus the cost of producing Q_r" measured by Q_r"A, separately.

Following Panzer and Willig (1981),[46] global economies of scope is said to exist in the two product case when following condition holds:

$$C(Q_y,Q_r) < C(Q_y,0) + C(0,Q_r) \tag{24}$$

---

[46]Also see Baumol, Panzer and Willig (1982), Bailey and Friedlaender (1982), and Nadiri (1982).

Baumol, Panzer and Willig (1982) define the degree of economies of scope in the two product case as:

$$SCOPE = (C(Q_y,0)+C(0,Q_r)-C(Q_y,Q_r))/C(Q_y,Q_r) \qquad (25)$$

If SCOPE < 0, then there are economies of scope which means there is a cost saving from producing the products jointly. If SCOPE > 0, then there are no economies of scope which indicates there are additional costs from joint production. Finally, if SCOPE = 0, then there are no additional costs or cost savings associated with joint production.

A test for the existence of local economies of scope involves the notion of cost complementarities. According to Baumol, Panzer and Willig (1982) a twice differentiable multiproduct cost function exhibits weak cost complementarities if:[47]

$$\delta^2 C(Q)/\delta Q_y \delta Q_r < 0 \qquad (26)$$

---

[47]See Baumol, Panzer and Willig (1982), p. 75, Definition 4B3.

Weak cost complementarities are a sufficient condition for economies of scope.[48]

One would expect at the micro level, there may exist economies of scope for mixed farming operations since the production of say livestock and feed use sharable inputs.[49] However, at the aggregate level given the diverse nature of aggregated outputs, one would expect the existence of economies of scope for Canadian agriculture to be highly unlikely.

2.5 Functional Specification

Given the results of section 2.1, it is clear that under certain circumstances, the economist may employ either the cost function or production function to equivalently and alternatively describe the technology of the firm. However, regardless of which approach is utilized, the econometrician, in order to empirically implement the analysis, must specify an explicit parametric functional form for either of the

---

[48]Baumol, Panzer and Willig (1982), p. 74, Proposition 4B1.

[49]Baumol, Panzer and Willig (1982) point out that the joint use of a factor of production to produce several products (sharable inputs) will lead to economies of scope.

aforementioned functions.[50,51] The purpose of this section is to provide a brief discussion of functional specification.

The choice of an explicit parametric specification should be guided by the "true" underlying production technology inferred from observation and economic theory. Ideally, the analyst would like economic theory and a set of data to determine the correct specification of the mathematical estimated form. The employment of this correct specification in conjunction with another set of data drawn from the same population would then be used for the estimation of the parameters of this specification.[52] Consequently, a series of summary measures would then be calculated or estimated supplemented by a sequence of

---

[50]Non-parametric approaches to the study of production have recieved some attention in economics. However, these methods have been exploited less systematically vis-a-vis parametric specification.

[51]Alternatively, one may specify a functional form for the derived input demand equations or various elasticity measures and then integrate to determine the specification of the production function or cost function.

[52]Theil (1978), p. 273 states,

> "When the observations are plentiful, a sensible approach is to divide them into three parts. One set of observations should be used for the specification of the mathematical form of the relation. The second set should be used for the estimation of the parameters of this specification, and the third for predictions based on the estimated equation to verify whether the specification selected is acceptable."

hypothesis testing. Therefore, the parameter estimates, the summary measures and the hypothesis testing would be generated by the set of observations and the true specification.

Unfortunately, problems with "impure" and limited data introduces elements of functional obscurity and ambiguity. Furthermore, economic theory gives little guidance with respect to a choice of an appropriate specification. Theory, typically, suggests general causal relationships such as in the present context that the cost function is a function of input prices and levels of output. However, theory does not indicate whether the function is linear, logarithmic, multiplicatitive, quadratic, polynomial or a more complicated mixed form. Dhyrmes et al. (1972) has stated:

> "Economic theory gives preciously few clues as to the functional forms appropriate to the specification of economic relationships, and the presence of random error terms in stochastically specified equations adds an additional element of functional ambiguity."

Accordingly, economists generally have either arbitrarily specified, a priori, an explicit functional form and treated this specification as a maintained (untested) hypothesis, or have utilized various statistical techniques to discriminate between alternate

model specifications.[53] A cursory examination of the empirical literature indicates the former approach has been more often employed than the latter.

In this case, the maintained hypothesis of an ad hoc functional specification, in part, influences, conditions and predetermines the parameter estimates and hence the summary measures. If the ad hoc specification is inconsistent with the true underlying function, then a specification error would have been introduced and the analysis and the subsequent policy recommendations formulated by the numerical estimates may be misleading. Kmenta (1971) has tersely stated:[54]

> "...by relying on the assumptions contained in the maintained hypotheses, we get results that are strictly conditional on these assumptions and do not hold without them. This elementary fact seems to be frequently forgotten in applied econometric research."

Moreover, specific hypothesis testing in empirical analysis depends on the validity of both the maintained hypothesis and the hypothesis under examination. Fuss,

---

[53]See Theil (1971, 1978) for a summary of model discrimination techniques.

[54]See Kmenta (1971), p. 136.

McFadden, and Mundlak (1978) states:[55]

> "...a test performed in the presence of an implausible maintained hypothesis may not be convincing, the results may be a consequence of the validity of the maintained hypotheses rather than of the primary hypothesis in which one is interested."

In light of these arguments, particular attention must be allocated to the choice of a function specification. In order to circumvent problems associated with the arbitrary specification of a specific functional form, the analyst seeking "proper" parameter estimates may use methods to:

1. discriminate among alternative feasible specifications which are non-nested,
2. discriminate among models which are nested in some generalized form or
3. approximate the "true" unknown function.

Recent literature seeking to analyze production technologies via the use of the primal or dual approach have popularly used either approximating analysis or the generalized form approach.[56] In general both techniques

---

[55]Fuss, McFadden, and Mundlak (1978), p. 223.

[56]Approximating analysis and the generalized form approach has also been used in terms of the theory of demand and the consumer.

use fewer maintained hypothesis than previous production studies and allowed for "flexibility" in modeling technology.

2.6 Flexible Functional Forms

A functional form characterized by flexibility may be defined as a parametric representation of a function which:

1. does not, a priori, constrain the parameter estimates and therefore the various summary measures beyond the confines imposed by neoclassical theory,
2. allows certain structural hypothesis such as separability for example to be testable rather than constituting them as a part of the maintained hypothesis,
3. expands the set of feasible alternative descriptions of technology which is possibly consistent with the true underlying specification.

The characteristic of flexibility allows one to minimize the number of explicit and implicit maintained hypotheses in comparison to the number that were heretofore necessary. Two common methods of deriving functional forms which possess the flexibility characteristic are approximating analysis and generalization of form. These two methods of generating

flexible characteristics need not be mutually exclusive.

An approximating form may be viewed as a parametric approximation of an arbitrary or true function where in fact the true function is unknown. The most popular type of approximation is the approximation of the second order. However, Lau (1974) has identified at least two different definitions of an second order approximation.

Following Lau (1974), suppose F(X) is the true specification of some production technology then Z(X) is a second order differential approximation if the image of Z(X) and its first and second order derivatives evaluated at a point such as X_0 is equal to the image, and the first and second order derivatives of the function being approximated. Contrarily, Z(X) is a second order numerical approximation to F(X) if Z(X) = F(X) evaluated at a point such as X_0 and for deviations of X from X_0 where the deviations between the approximating and true function is bounded by higher order terms.

These approximating functions have been referred to as flexible functional forms. A Taylor series expansion of the second order at an arbitrary evaluation point is a second order differential approximation. The quadratic, generalized Leontief (Diewert,1971), generalized quadratic of the mean order r (Denny, 1974), the translog function (Christensen, Jorgenson and Lau, 1973), and the

generalized Box-Cox (Berndt and Khaled, 1979) are examples of flexible functional forms having the Taylor series interpretation . The generalized Cobb-Douglas (Diewert, 1973) and the mean order of two (Diewert, 1974a) are flexible functional forms which do not have the Taylor series interpretation. The former approach to functional specification has been much more popular than the latter.

If one uses a flexible functional form with the Taylor series interpretation then the properties of the true function which are preserved only hold at the point of approximation or evaluation. Nonetheless, the use of flexible functional forms allows one to test particular hypotheses which were previously maintained such as separability.

Alternatively, the analyst can expand the feasible set of alternative functional specifications and incorporate the characteristic of flexibility by utilizing generalized forms. A generalized form is a parametric specification of some given form such that the generalized form contains the given form as a subset. Suppose F(X) is some given form, then G(F(X)) is a generalized form such that, depending on the values of the parameters in G(F(X)), it will collapse into F(X).

Generalized forms can be derived by numerous methods such as the introduction of additional parameters or the

transformation of variables. As an example, suppose one is given a Cobb-Douglas form of a production function with a single output y and inputs labor (L) and capital (K) such as:

$$Y = AL^{a_l}K^{a_k} \tag{27}$$

or

$$\ln y = a + a_l \ln L + a_k \ln K$$

One can generalized the Cobb-Douglas form of (27) to:[57]

a. the CES production function of the form as:

$$y = A\{a_l L^{-B} + (1 - a_l)K^{-B}\}^{-1/B}$$

b. the transcendental production function of the form:

$$\ln y = a + a_l \ln L + a_k \ln K + a_l' L + a_k' K$$

c. the Zellner-Revankar production function of the form:

$$\ln y + cy = a + a_l \ln L + a_k \ln K$$

d. the Nerlove-Ringstad production function of the form:

$$(1 + c\ln y)\ln y = a + a_l \ln L + a_k \ln K$$

e. the translog production function of the form:

$$\ln y = a + a_l \ln L + a_k \ln K + a_{lk} \ln L \ln K$$

---

[57]See Intriligator (1978) and Nadiri (1982) for a similar listing.

$$+ b_l(\ln L)^2 + b_k(\ln K)^2$$

f. generalizations of the CES function such as the VES

g. generalizations of the translog such as the hybrid translog

h. the classical Box-Cox production function where only the dependent variables are transformed

i. the Box-Tidwell production function where only the independent variables are transformed

j. the extended Box-Cox production function where all variables are transformed by the same power

h. the generalized Box-Cox production function where all variables are transformed by different powers.

Depending on the particular values of the above functions the Cobb-Douglas may be obtained.

One can clearly observe the translog function and the generalized Box-Cox may be viewed as either an approximating function or a generalized function. If the translog function is treated as an approximating function then the parameter estimates obtained only hold at the point of evaluation. Furthermore, valid hypothesis testing can only be done at the point of evaluation. Questions arise as to what characteristics of the true function are captured by the approximating function other than the preservation of the equality of the image, the first and second order derivatives at the point of

approximation.

If one treats the translog function as a generalized function then the analyst is treating the translog specification as an exact function such that the true underlying structure is in fact translog. This interpretation may lead to a specification error particularly if the level of generalization is relatively low.

The distinction between the approximating and exact interpretation of a functional form such as the translog is important since the approximation to a separable function need not be separable.[58] Several studies have typically used a flexible functional form stating its virtues as a second order approximation to an twice differentiable arbitrary function and then treated the function as an exact one employing it as a generalized function.[59] Guilkey and Lovell (1978) have stated:[60]

> "...these forms have been used to represent preferences and technology in two different ways.

---

[58]See Blackorby, Primont and Russell (1978), p. 297.

[59]Ibid.

[60]For others who make the distinction between exact and approximating functions see Burgess (1975), Blackorby, Primont and Russell (1974), Denny and Fuss (1977), and Lau (1974) for example.

> One approach ...treats these forms as second order local approximations to arbitrary twice differentiable functions that themselves represents preferences or technology. ...the alternative approach of treating flexible forms as exact representations of the structure of preferences or technology."

In order to be consistent, the analyst should treat whatever functional form that is employed as either an approximating or exact form. However, if one is concerned with the global properties of a production technology, then the analyst should not use the flexible functional form as an approximating form. Since the author is interested in both the global and local properties of the production technology, the perspective of this thesis will be to view whatever functional form is employed as an approximating function in most cases and as an exact function when global tests are conducted. Moreover, in order to avoid confusion on the part of the reader, it will be clearly stated what interpretation of the functional form is being used

CHAPTER III: THE MODEL

This chapter develops the specific model which will be used to estimate the production technology of Canadian agriculture. Section 3.1 introduces the the nonhomothetic joint multiple output, multiple input translog cost function. The cost share and revenue share equations are also presented.

Section 3.2 develops several alternative sets of hypotheses which will be used to test the global properties of the production structure of Canadian agriculture. Restrictions for two types of global homotheticity, three alternative versions of homogeneity and four alternative specifications of global linear homogeneity are presented.

The functional form of the various elasticities are presented in Section 3.3. Given the multiproduct translog cost function, the formulae for the Allen-Uzawa partial elasticity of factor substitution, the partial cross price and own price elasticities of factor demands, the output elasticities of factor demands and the marginal rate of product transformation are presented.

Finally, section 3.4 presents the tests and measures

for overall economies of scale and economies of scope, given the translog multiple input, multiple output cost function.

3.1 The Multiple Input, Multiple Output Translog Cost Function

In order to empirically implement the analysis of the preceding chapter, one requires a functional specification of the cost function. The functional form should be sufficiently flexible to allow the analyst to test various hypotheses econometrically without imposing unduly necessary restriction upon the structure of the production technology in question.

Treating the functional specification of the cost function as a second order Taylor series expansion, the multiple input, multiple output agriculture cost function may be written in the translog form. It is assumed the eastern and western Canadian agriculture sectors can be represented by regional multiple output, multiple input

cost function of the translog form[61] as:

$$\begin{aligned}\ln C(P,Y) = a_0 &+ \sum_i a_i \ln P_i + 1/2\sum_i\sum_j b_{ij} \ln P_i \ln P_j \\ &+ \sum_y q_y \ln Q_y + 1/2\sum_y\sum_r q_{yr} \ln Q_y \ln Q_r \\ &+ \sum_i\sum_y g_{iy} \ln P_i \ln Q_y \qquad (1)\end{aligned}$$

where:

$i, j = 1, 2, \ldots, n.$

$y, r = 1, 2, \ldots, m.$

The imposition of symmetry requires:

$b_{ij} = b_{ji}$ for all i and j, $i \neq j$

$q_{yr} = q_{ry}$ for all r and y, $y \neq r$

Homogeneity of degree one in input prices and symmetry requires the following within equation restrictions on (1):

$\sum_i a_i = 1$ $i, j = 1, 2, \ldots, n.$

$b_{ij} = b_{ji} = 0$ and $i \neq j$

$\sum g_{iy} = 0$ $y = 1, 2, \ldots, m.$

---

[61] An alternative formulation of the cost function is of the hybrid translog form where the output variables are transformed by the Box-Cox metric. Although this specification has become widely used, particularly in the regulation literature, it could not be employed in the present study due to the much higher cost of the estimation, the lack of an adequate sample size, and the unavailability of a subroutine in the Shazam program to transform each variable separately when the variables are multiplicative.

Partially differentiating ln C(P,Y) with respect to ln P_i and invoking Shepard's lemma yields the cost share equations of the translog cost function represented by:

$$S_i = \delta \ln C(P,Y)/\delta \ln P_i$$
$$= a_i + \sum_j b_{ij} \ln P_j + \sum_y g_{iy} \ln Q_y \qquad (2)$$

where:

$i, j = 1, 2, \ldots, n.$

$y = 1, 2, \ldots, m.$

Since the cost function is homogeneous of degree one in input prices, then each cost share equation must be homogeneous of degree zero in input prices which imply the following within equation restriction:

$$\sum_j b_{ij} = 0 \qquad i, j = 1, 2, \ldots, n.$$

Furthermore, since the cost shares must sum to one the following across the equation restrictions are imposed on the n cost share equations:[62]

$$\sum_i a_i = 1$$

---

[62] Alternatively, these across the equation restrictions are a result of linear homogeneity in input prices of the cost function and the across equation equality of parameters in the cost function and the share equations.

$$\sum_j b_{ij} = 0 \qquad i, j = 1, 2, \ldots, n.$$
$$\sum_i g_{iy} = 0 \qquad y = 1, 2, \ldots, m.$$

Assuming cost minimization and marginal cost pricing, partially differentiating ln C(P,Y) with respect to ln Q_y yields the revenue share equation of the form:

$$R_y = \delta \ln C / \delta \ln Q_y$$
$$= q_y + \sum_r q_{yr} \ln Q_r + \sum_i g_{iy} \ln P_i \qquad (3)$$

where:

$i = 1, 2, \ldots, n.$

$y, r = 1, 2, \ldots, m.$

Since the industry is not constrained to zero economic profits, the revenue share equations are not constrained to sum to one. However, since the cost function is homogeneous of degree one in input prices, this implies the revenue share equations have the following within the equation restriction:

$$\sum_i g_{iy} = 0 \qquad i = 1, 2, \ldots, n.$$

The system of equations (1), (2) and (3) with the appropriate set of independent restrictions[63] constitute

---

[63]Not all of the above restrictions are independent. The appropriate set of independent restrictions is fully specified in the next chapter.

the full cost system.

3.2 Restrictions for Alternative Structure

Given the nonhomothetic joint cost structure of the translog form given by equation (1), various alternative structures of the production technology can be tested by examining certain parameter restrictions of the cost function.

The translog dual multiproduct cost function (1) represents a weakly homothetic production function if:[64]

$$q_r g_{iy} = q_y g_{ir}$$

where:

$$i = 1, 2, 3, \ldots, n.$$

$$y, r = 1, 2, 3, \ldots, m. \quad y \neq r$$

holds. This suggests input prices in the cost function are weakly separable from output levels. Denny and May (1978b) have shown that the simultaneous imposition of weak homotheticity of the production structure and linear homogeneity in input prices of the cost function necessarily imply a strongly homothetic production

---

[64]See Denny and Pinto (1978), p. 256.

structure. As a result, the weakly homothetic case is not independently tested in the empirical section.[65]

However, it is well known that every cost function which is homogeneous in output levels must always be homothetic. The translog cost function is homogeneous of degree $\sum q_y$ in outputs if the following restrictions hold:[66]

$$\sum_{y} q_{yr} = 0 \qquad y, r = 1, 2, \ldots, m.$$
$$\sum_{y} g_{iy} = 0 \qquad i = 1, 2, \ldots, n.$$

If the above holds, the production structure must be homothetic. Furthermore, the production structure is homogeneous of degree one if the translog cost function satisfies the following restrictions:[67]

$$\sum_{y} q_{yr} = 0 \qquad y, r = 1, 2, \ldots, m.$$
$$\sum_{y} g_{iy} = 0 \qquad i = 1, 2, \ldots, n.$$
$$\sum_{y} q_{y} = 1$$

The production function is said to be strongly

---

[65] It is unclear if Denny and May (1978b) are correct. Regardless, the Shazam program exmployed in estimation cannot handle nonlinear restrictions.

[66] Brown, Caves and Christensen (1979), p. 259.

[67] Ibid.

homothetic if the cost function is a strongly separable function of input prices and output levels (Denny and Fuss, 1977). For the translog form of the cost function, the technology is said to be strongly homothetic if the following restriction holds:[68]

$$g_{iy} = 0 \qquad i = 1, 2, \ldots, n.$$
$$\qquad\qquad y = 1, 2, \ldots, m.$$

Moreover, the cost function is strongly homothetic and weakly homogeneous if:[69]

$$g_{iy} = 0 \qquad i = 1, 2, \ldots, n.$$
$$\sum_{y} q_{yr} = 0 \qquad y, r = 1, 2, \ldots, m.$$

holds. In addition, the translog cost function is linear homogeneous in outputs if the following restrictions are satisfied:

$$g_{iy} = 0 \qquad i = 1, 2, \ldots, n.$$
$$\sum_{y} q_{yr} = 0 \qquad y, r = 1, 2, \ldots, m.$$
$$\sum_{y} q_{y} = 1$$

---

[68] Brown, Caves and Christensen (1979), p. 259 and Gillen and Oum (1983).

[69] Since there exists at least two types of homotheticity, then there exists several types of homogeneity.

Alternatively, given the fact the translog function tests empirically to be strongly homothetic, one can identify an alternative set of restrictions which is sufficient for a homogeneous translog function. The translog cost function represents a strongly homothetic and strongly homogeneous production structure if:

$$g_{iy} = 0 \qquad i = 1, 2, \ldots, n.$$
$$q_{yr} = 0 \qquad y, r = 1, 2, \ldots, m.$$

holds.

Furthermore, the translog cost function is homogeneous of degree one in outputs when the restrictions:

$$g_{iy} = 0 \qquad i = 1, 2, \ldots, n.$$
$$q_{yr} = 0 \qquad y, r = 1, 2, \ldots, m.$$
$$\sum_{Y} q_y = 1$$

are satisfied.

The Cobb-Douglas cost function is a subset of the translog cost function. The Cobb-Douglas cost function is strongly homothetic and strongly homogeneous. The translog function is log-linear if the following set of restrictions hold:[70]

70See Gillen and Oum (1983).

$$g_{iy} = 0$$

$$q_{yr} = 0 \qquad y, r = 1, 2, \ldots, m.$$

$$b_{ij} = 0 \qquad i, j = 1, 2, \ldots, n.$$

Consequently, the translog cost function will approximate the linear homogeneous Cobb-Douglas cost function if:

$$g_{iy} = 0$$

$$q_{yr} = 0 \qquad y, r = 1, 2, \ldots, m.$$

$$b_{ij} = 0 \qquad i, j = 1, 2, \ldots, n.$$

$$\sum_{Y} q_{y} = 1$$

Clearly one can observe all the global linear homogeneity tests are nested within the global homogeneity tests and the homotheticity tests. Figure 3 presents a summary of the structure of tests to be conducted on the translog cost function to determine the empirically validity of global homotheticity, homogeneity and linear homogeneity for Canadian agriculture.

Moreover, since there exists a large number of alternative restrictions to be statistically tested, a summary of the restrictions for global homotheticity, homogeneity and linear homogeneity is provided in Figure 4. All of restrictions specified in Figure 4 will be empirically tested in the next chapter.

Figure 3: Structure of Tests for Homotheticity and Homogeneity

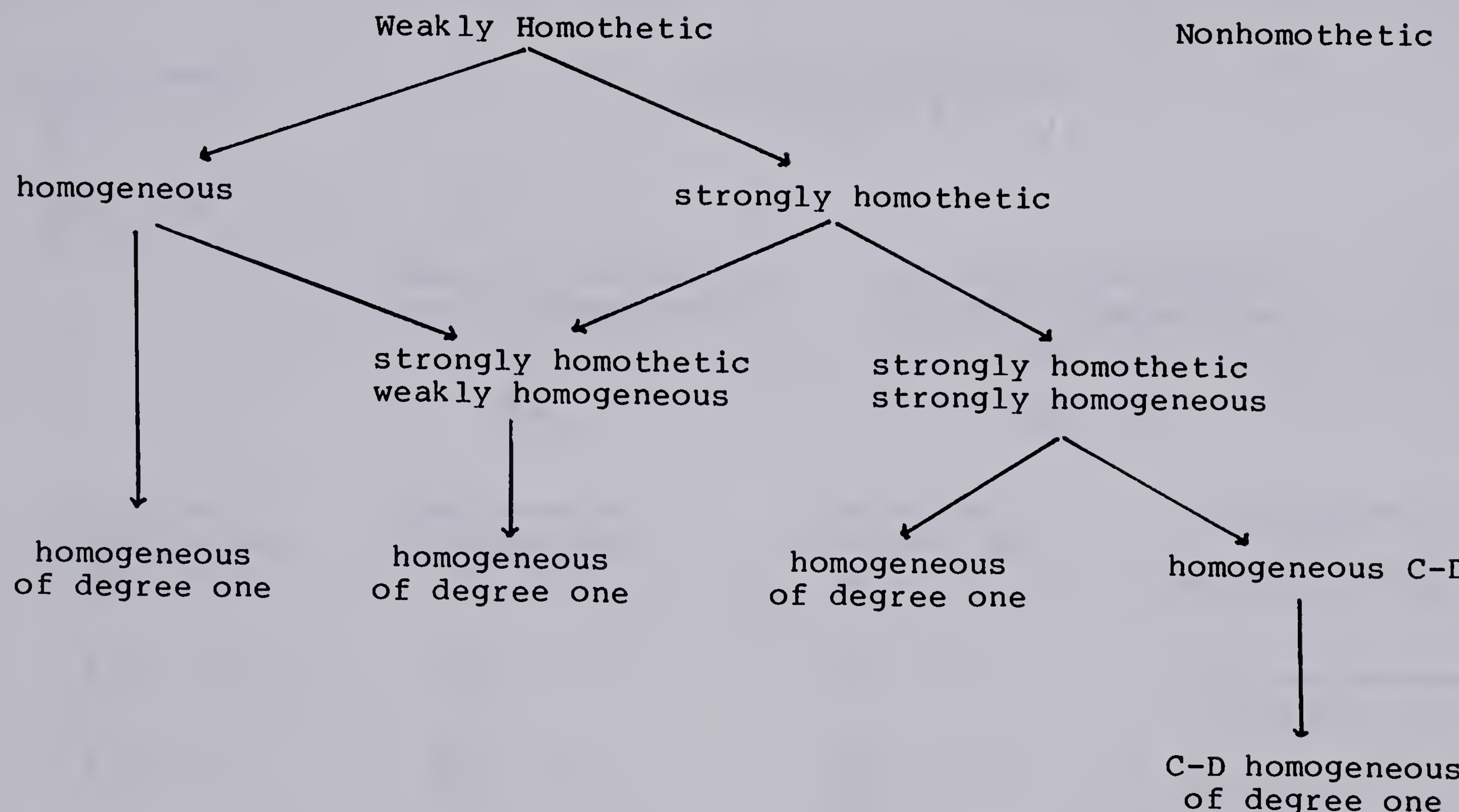


Figure 4: Restrictions on the Translog Cost Function for Homotheticity and Homogeneity

	Weakly Homothetic $q_r g_{iy} = q_y g_{ir}$		Nonhomothetic
homogeneous $\sum_y g_{iy} = 0$ $\sum_y q_{yr} = 0$		strongly homothetic $g_{iy} = 0$	
	strongly homothetic weakly homogeneous $g_{iy} = 0$ $\sum_y q_{yr} = 0$	strongly homothetic strongly homogeneous $g_{iy} = 0$ $q_{yr} = 0$	
homogeneous of degree one $\sum_y g_{iy} = 0$ $\sum_y q_{yr} = 0$ $\sum_y q_y = 1$	homogeneous of degree one $g_{iy} = 0$ $\sum_y q_{yr} = 0$ $\sum_y q_y = 1$	homogeneous of degree one $g_{iy} = 0$ $q_{yr} = 0$ $\sum_y q_y = 1$	homogeneous C-D $g_{iy} = q_{yr} = b_{ij} = 0$ C-D homogeneous of degree one $g_{iy} = q_{yr} = b_{ij} = 0$ $\sum_y q_y = 1$

3.3 Functional Forms of the Summary Measures

Once alternative structures of the production technology have been tested, the parameter estimates employed will originate from the empirically validated model. These parameter estimates in conjunction with the fitted cost function, cost share equations, and revenue share equations will be utilized to derive parametric values of the summary measures discussed in Section 2.3.

Berndt and Wood (1975) and Binswanger (1974b) have shown that the Allen-Uzawa partial elasticity of substitution between factor i and j given the translog cost function can be written as:

$$AUES_{ij} = (b_{ij}/S_iS_j) + 1 \qquad \text{for all } i \neq j \tag{4}$$

$$AUES_{ii} = (b_{ii} + S_i^2 - S_i)/S_i^2 \qquad \text{for all } i \tag{5}$$

The value of AUES depends upon the values of the fitted cost shares and as a result vary with relative changes in the cost shares. One can test alternative structure of the cost function by examining $AUES_{ij}$ since for the Cobb-Douglas $AUES_{ij} = 1$, the Leontief $AUES_{ij} = 0$

and for the CES form all $AUES_{ij}$ are equal.[71]

The translog cost function yields the following formula for the partial cross price and own price elasticities of factor demands (Berndt and Wood, 1975):

$$E_{ij} = (b_{ij} + S_i S_j)/S_i \qquad \text{for all } i \neq j \tag{6}$$

$$E_{ii} = (b_{ii} + S_i^2 - S_i)/S_i \qquad \text{for all } i \tag{7}$$

Using the results of the Allen-Uzawa partial elasticities of substitution of (4) and (5), one can calculate the cross and own price elasticities of factor demands for the translog cost function by:[72]

$$E_{ij} = S_j \cdot AUES_{ij} \qquad \text{for all } i \neq j \tag{8}$$

$$E_{ii} = S_i \cdot AUES_{ii} \qquad \text{for all } i \tag{9}$$

The output elasticity of factor demands derived from

---

[71]See Denny and May (1978b), p. 311.

[72]This relationship was shown orginally between AES and E by Allen (1938).

the translog cost function can be expressed as:[73]

$$E_{iy} = (g_{iy}/S_i) + R_y \qquad \text{for all } y, i \tag{10}$$

It should be noted if the cost function is strongly separable in input prices from outputs such that $g_{iy} = 0$, then the output elasticities of factor demands depends on the level of outputs only and is independent of factor input prices.

The marginal cost of producing y, given the translog cost function, can be expressed as (Darrough and Heineke, 1978):

$$MC_y = (q_y + \sum_r q_{yr} \ln Q_r + \sum_i g_{iy} \ln P_i)(C/Q_y) \tag{11}$$

where:

$i = 1, 2, 3, \ldots, n.$

$y, r = 1, 2, 3, \ldots, m.$ and $y \neq r$

Finally, for the translog multiproduct cost function, the marginal rate of transformation of product y for product r can be determined by the expression suggested by Darrough and Heineke (1978) written as:

73See Appendix B for the derivation of this final form.

$$MRT_{yr} = MC_r/MC_y$$

$$= \frac{(q_r + \sum_{r} q_{yr} \ln Q_y + \sum_{i} g_{ir} \ln P_i)Q_y}{(q_y + \sum_{r} q_{yr} \ln Q_r + \sum_{i} g_{iy} \ln P_i)Q_r} \quad (12)$$

where:

$i = 1, 2, 3, \ldots, n.$ and $y \neq r$.

All of the summary measures presented in this section will be estimated using the parameter estimates and the fitted cost, cost share and revenue data.

3.4 Economies of Scale and Economies of Scope

The use of the translog multiproduct cost function results in limitations with respect to particular tests of economies of scale and economies of scope. The tests and measures developed in the previous chapter for global economies of scope and global product-specific economies of scale require the cost function to be clearly defined for positive output vectors where an element, say Q_y, is set to zero. The translog cost function is not well defined for zero output levels since the logarithm of zero is undefined. As a result, tests for global economies of scope and global product-specific economies of scale are not conducted.

Caves, Christensen and Tretheway (1980) suggest the use of the generalized translog cost function, where the output variables are transformed by a Box-Cox metric,[74] to test for global economies of scope and product-specific economies of scale. Gillen and Oum (1983) point out, even though technically the generalized translog cost function can be used, the calculations of the above measures requires extrapolation of the estimated cost function well beyond the range of observed data which may led to questionable empirical conclusions. As a result, Gillen and Oum (1983) conduct tests for economies of scope which are local in nature.

In the previous chapter, it was stated that cost complementarity or jointness was a sufficient condition for local economies of scope. The test statistic for cost complementarity in the multiproduct translog cost model can be written as:[75]

$$CC_{yr} = \frac{C}{Q_y Q_r}\left[\frac{\delta \ln C}{\delta \ln Q_y}\frac{\delta \ln C}{\delta \ln Q_r} + q_{yr}\right] \quad y \neq r. \tag{13}$$

which in the two product case becomes:

---

[74]The Box-Cox transformation of output Q_y is $(Q_y^{\lambda} - 1)/\lambda$.

[75]See Fuss and Waverman (1978), Kiss, Karabadjian, and Lefebvre (1981) and Gillen and Oum (1983).

$$CC_{yr} = \frac{C}{Q_y Q_r} ([q_y + \sum_r q_{yr} \ln Q_r + \sum_i g_{iy} \ln P_i] \cdot [q_r + \sum_y q_{yr} \ln Q_y + \sum_i g_{ir} \ln P_i] + q_{yr}) \qquad i = 1, 2, \ldots, n. \tag{14}$$

or substituting the definition of revenue shares the above can be written as:

$$CC_{yr} = \frac{C}{Q_y Q_r} (R_y \cdot R_r + q_{yr}) \qquad y \neq r \tag{15}$$

Since the value of CC_{yr} depends on the data, it has been common to scale the data at a particular point to calculate a value of CC_{yr} independent of the data (Fuss and Waverman (1978), Kiss, Karabadjian, and Lefebvre (1981) and Gillen and Oum (1983)). At the point of expansion after the data has been scaled such that $Q_y = P_i = 1$, then the test statistic for jointness becomes:

$$CC_{yr} = q_y q_r + q_{yr} \tag{16}$$

If $CC_{yr} < 0$ holds, then there exists cost complementarity or jointness in production which implies the existence of local economies of scope.

In essence, there are two methods for determining the existence of cost complementarity and hence local economies of scope. First, one can check the sign of

(13), (14), (15) or (15) at all data points (Gillen and Oum, 1983). If the sign of CC_{yr} is negative at all data points, then that is sufficient for the existence of economies of scope. Alternative, using (16), one can test for the null hypothesis of no cost complementarity by setting CC_{yr} to zero at the expansion. If this null hypothesis is rejected and if the alternative is true then there are local economies of scope.[76]

In addition to a measure for economies of scope using the translog multiproduct cost function, one can calculate a measure for local overall economies of scale. There exist two methods by which local overall economies of scale can be tested. The first measure involves calculating the inverse of the sum of the cost elasticities with respect to outputs. Given the translog multiproduct cost function, a measure of local overall economies of scale can be expressed as:[77]

$$ES = \{\sum_{y}(q_y + \sum_{r} q_{yr} \ln Q_r + \sum_{i} g_{iy} \ln P_i)\}^{-1} \qquad (17)$$

---

[76] There are two methods associated with testing (16). Fuss and Waverman (1978) scale the data and use the twice the logarithm of the likelihood ratio test while Kiss, Karabadjian and Lefebvre (1981) construct a confidence interval around the right hand side of (16) to see if $CC_{yr} = 0$ falls within the confidence interval.

[77] See Bailey and Friedlaender (1982).

An alternative method to determine the degree of local overall economies of scale involves scaling the data at the point of approximation such that $P_i = Q_y = 1$ and hence (17) reduces to:[78]

$$ES = \left(\sum_{Y} q_Y\right)^{-1} \tag{18}$$

If ES > 1 then there is local overall economies of scale. Alternatively, if ES < 1 then there exist local overall diseconomies of scale. Finally, if ES = 1 then there is no local overall economies or diseconomies.

In the empirical chapter of this thesis, I will conduct the statistical analysis to test for local economies of scope and local overall economies of scale.

---

[78]See Gillen and Oum (1983).

CHAPTER IV: ECONOMETRIC ESTIMATION

This chapter presents the empirical results from the estimation of the complete nonhomothetic, joint, multiple input, multiple output translog cost model outlined in Chapter Three. Section 4.1 presents a discussion of the data set utilized in the estimation. Section 4.2 discusses the methodology employed in the estimation of the complete model. In addition, regional dummy variables to capture the interregional differences are introduced.

In section 4.3, the empirical results of the global tests of the production structure are presented. In particular, the statistical conclusions with respect to global homotheticity, homogeneity, linear homogeneity and log linearity are discussed.

The parameter estimates of the general nonhomothetic, joint model are presented in section 4.4. Moreover, the estimated values of the Allen-Uzawa partial elasiticities of factor substitution, the partial cross price and own price elasticities of factor demands, the output elasticities of factor demands and the marginal rate of product transformation are presented and discussed in this section.

Finally, section 4.5 presents and discusses the empirical results of the tests conducted for local economies of scope and local overall economies of scale.

4.1 Data

The data necessary to estimate the complete cost model must be comprised of input price indexes and cost share data for n inputs, output indexes and revenue share data for m outputs, and total cost data. Given the complete translog cost model of n inputs and m outputs, the number of parameters which need to be estimated is $(1/2)(m + n)(3 + m + n) + 1$.[79] Consequently, the sample size places upper limits on the magnitude of m and n such that the model can be empirically implemented.

As a result of data limitations, it was decided to use a four input, two output cost model. The four inputs are comprised of land, labor, capital and materials while the two outputs consist of field crops and livestock/animal products.

The data employed in estimation originates from two

---

[79] This is after symmetry, linear homogeneity in input prices, and the adding up condition has been imposed. See Ray (1982).

major sources; Agriculture Canada[80] and Statistics Canada. The data set is comprised of pooled time series and cross sectional data for Western and Eastern Canada for the time period 1961 to 1979.

Land is one of the most important variables in determining the characteristics of Canadian agriculture. What is required for the purpose at hand is a measure of the rental price of land reflecting the costs incurred, either directly or indirectly, from a flow of services derived from a given stock of land. The rental price index of land and the value of the annual flow of services from land for Eastern and Western Canada was calculated by Agriculture Canada.

The labor price index and the value of labor input was provided by Agriculture Canada. The hired labor wage rate was used to construct a labor price index and it was assumed this wage was paid to hired, operator, and family labor. The wage rate for Eastern Canada is a weighted average of the hourly agriculture wage rate for each Eastern province. A similiar procedure was employed to determine the Western Canada agriculture wage rate. The total regional agriculture wage bill reflects explicit and

---

80This data set was used in Lopez and Tung (1982). My thanks to Agriculture Canada and in particular, Drs. Lopez, Tung and Nararayan.

implicit payments to hired, operator and family labor.

The regional capital price index provided by Agriculture Canada is derived by a weighted average of machinery and equipment, buildings and fencing, and animal stock price indexes using 1971 cost share regional weights. The regional capital price indexes represents the rental price of annual per unit flows of services originating from the existing capital stock in each year. Aggregating the value of the flow of services from each subcomponent yields the value of the flow of services from capital.

The material price index is a weighted average of the energy,[81] energy based,[82] feed, seed, livestock expense,[83] other crop expenses[84] and miscellaneous expense[85] price indexes using 1971 cost share weights. The expenditures on materials was determined by summing all subcomponents.

---

[81]Includes fuel and electricity.

[82]Comprised of fertilizer, lime, and agriculture chemicals.

[83]Consists of livestock purchases, registration fees, and veterinary expenses.

[84]Composed of nursary stock, irrigation, containers and twine.

[85]Includes custom work, insurance, and other supplies and services.

The total annual cost of production for each region was determined by summing the annual land, labor, capital and material expenses for each region. The cost share data was calculated by dividing the expenditures on each input by the total cost of production.

Regional annual total revenues from field crops is comprised of the summation of regional revenues from wheat,[86] oats, barley, rye, flaxseed, rapeseed, soybeans, corn, sugar beets, potatoes, fruits, vegetables, tobacco, and other crops. The regional quantity index was constructed by dividing the total regional revenues from field crops by the regional field crop price index[87] and then transforming the resultant values into an output quantity index.

Annual total revenues from livestock and animal products was found by aggregating regional revenues from cattle, calves, hogs, dairy products,[88] poultry, eggs, sheep, lambs and other livestock and animal products. The regional quantity index for livestock and animal products

---

[86]Includes Canadian Wheat Board payments and net cash advance payment under the provisions of the Prairie Advance payment Act and Western Grain Stabilization Act.

[87]Calculated as a weighted average of the Canadian field crop price index and the regional agriculture price index.

[88]Including dairy supplementary payments.

was constructed using the procedure similar to the one indicated above.

The revenue shares, assuming competition, was calculated by dividing total regional revenues of each aggregate output by the total cost of production.

In brief recapitulation, Table 3 presents a list of definitions of the varibles employed. All index data used in estimation was normalized to one for 1971.

4.2 Estimation Methodology

The cost function, cost share equations and revenue share equations were estimated simultaneously. Since the cost shares must add up to one, their disturbance terms sum to zero. Consequently, one of the cost share equations was deleted to preserve the nonsingularity of the covariance matrix. The material share equation was deleted. Maximum likelihood estimates are invariant to the equation deleted.

The full system, comprised of the cost function, n - 1 share equations and m revenue share equations, was estimated by an iterative estimation technique for Zellner´s seemingly unrelated equations which ensures that maximum likelihood estimates are obtained if the

Table 3: Definitions of Variables

Variable		Definition
P_L	Land input price index	Index of the rental price of land
P_N	Labor input price index	Index of the hourly wage rate
P_K	Capital input price index	Index of the rental price of capital
P_M	Material input price index	Index of the price of all other materials
Q_A	Field crop quantity index	Index of the output of all field crops
Q_F	Livestock and animal products quantity index	Index of the output of all livestock and animal products
S_L	Land cost share	Estimated cost of the flow of land services divided by total costs
S_N	Labor cost share	Estimated cost of operator, family and hired labor divide by total costs
S_K	Capital cost share	Estimated cost of the flow of capital services divided by total costs
S_M	Material cost share	Total expenditures on materials divided by total costs
R_F	Field crop revenue share	Total revenues recieved from all field crops divided by total cost
R_A	Livestock and animal products revenue share	Total revenues recieved from all livestock and animal products divided by total costs

covariance matrix converges.[89]

Actually, there exists six alternative variants of the model which could have been used in estimation. Given the cost model, one could estimate the parameters of the production technology by:

1. the cost function alone with linear homogeneity in input prices imposed.[90]

2. the cost function and n - 1 cost share equations with the adding up condition imposed.[91]

3. the n -1 cost share equations with the adding up condition imposed.[92]

4. the cost function and the m revenue share equations with homogeneity of degree one in input prices imposed.[93]

5. the n - 1 cost share equations and the m revenue share equations with the adding up condition imposed.[94]

---

[89]The convergence criteria was set at .05 with the maximum number of iterations set at 50.

[90]There may exists problems with multicollinearity.

[91]This method appears to be the most popular method of late but does not use all of the information of the complete model.

[92]Not all of the parameters of the cost function can be estimated. In particular, this method does not provide direct values for a_0, q_y, and q_{yr}.

[93]The author is not aware of any study which has employed this approach.

[94]The intercept term of the cost function cannot be directly estimated by this approach.

6. the cost function, the n - 1 cost share equations and the m revenue share equations with the adding up condition imposed.[95]

Preliminary regression results indicate the cost function, the n - 1 cost share equations and the m revenue share equations with the adding up condition imposed provided the most suitable econometric results.[96]

Dummy variables were introduced into each equation to capture the quantitative differences in regional production technology characteristics. The dummy variable, D, had the value of zero for Western Canada and one for Eastern Canada. Following Binswanger (1974b), Fuss (1977), Griffen and Gregory (1978), Lopez and Tung (1982) and MacRae and Webster (1980) the dummy variables were entered in additive form in each equation.[97] In addition, Hicks´ neutral technological change was assumed.

---

[95]One would expect the inclusion of the cost share and revenue share equations along with the cost function would increase the available degrees of freedom and improve statistical precision since the parameters of the cost and revenue share equations are a subset of the cost function.

[96]This is contrary to the findings of Guilkey and Lovell (1980). It appears to be the case as n and m become larger, the estimates provided by (1) degenerate vis-a-vis (6). Ray (1982) also found the full cost model provided better estimates.

[97]More correctly, each regional model should have been estimated separately. However, due to data limitations such a procedure was not possible.

The econometric model estimated had the following form.[98] The multi-input/multi-output cost function was written as:

$$\begin{aligned}\ln C = a_0 &+ a_L \ln P_L + a_N \ln P_N + a_K \ln P_K \\ &+ a_M \ln P_M + b_{LN} \ln P_L \ln P_N + b_{LK} \ln P_L \ln P_K \\ &+ b_{LM} \ln P_L \ln P_M + 1/2\, b_{LL} \ln P_L \ln P_L \\ &+ 1/2\, b_{NN} \ln P_N \ln P_N + b_{NK} \ln P_N \ln P_K \\ &+ b_{NM} \ln P_N \ln P_M + b_{KM} \ln P_K \ln P_M \\ &+ 1/2\, b_{KK} \ln P_K \ln P_K + 1/2\, b_{MM} \ln P_M \ln P_M \\ &+ q_F \ln Q_F + q_A \ln Q_A + q_{FA} \ln Q_F \ln Q_A \\ &+ 1/2\, q_{FF} \ln Q_F \ln Q_F + 1/2\, q_{AA} \ln Q_A \ln Q_A \\ &+ g_{LF} \ln P_L \ln Q_F + g_{NF} \ln P_N \ln Q_F \\ &+ g_{KF} \ln P_K \ln Q_F + g_{MF} \ln P_M \ln Q_F \\ &+ g_{LA} \ln P_L \ln Q_A + g_{NA} \ln P_N \ln Q_A \\ &+ g_{KA} \ln P_K \ln Q_A + g_{MA} \ln P_M \ln Q_A + d_C D \qquad (1)\end{aligned}$$

The cost share equations used were:

$$\begin{aligned}S_L = a_L &+ b_{LL} \ln P_L + b_{LN} \ln P_N + b_{LK} \ln P_K \\ &+ b_{LM} \ln P_M + g_{LF} \ln Q_F + g_{LA} \ln Q_A + d_L D\end{aligned}$$

$$\begin{aligned}S_N = a_N &+ b_{NN} \ln P_N + b_{LN} \ln P_L + b_{NK} \ln P_K \\ &+ b_{NM} \ln P_M + g_{NF} \ln Q_F + g_{NA} \ln Q_A + d_N D\end{aligned}$$

$$\begin{aligned}S_K = a_K &+ b_{KK} \ln P_K + b_{LK} \ln P_L + b_{NK} \ln P_K \\ &+ b_{KM} \ln P_M + g_{KF} \ln Q_F + g_{KA} \ln Q_A + d_K D \qquad (2)\end{aligned}$$

---

[98]Symmetry is imposed throughout the remainder of the presentation.

The revenue share equations have the form:

$$R_F = q_F + q_{FA} \ln Q_A + q_{FF} \ln Q_F + g_{LF} \ln P_L$$
$$+ b_{NF} \ln P_N + b_{KF} \ln P_K + b_{MF} \ln P_M + d_F D$$

$$R_A = q_A + q_{FA} \ln Q_F + q_{AA} \ln Q_A + g_{LA} \ln P_L$$
$$+ b_{NA} \ln P_N + b_{KA} \ln P_K + b_{MA} \ln P_M + d_A D \quad (3)$$

Furthermore, the following within and across equation restrictions were imposed on the model:[99]

$$a_L + a_N + a_K + a_M = 1$$
$$b_{LL} + b_{LN} + b_{LK} + b_{LM} = 0$$
$$b_{LN} + b_{NN} + b_{NK} + b_{NM} = 0$$
$$b_{LK} + b_{NK} + b_{KK} + b_{MM} = 0$$
$$b_{LM} + b_{NM} + b_{KM} + b_{KK} = 0$$
$$g_{LF} + g_{NF} + g_{KF} + g_{MF} = 0$$
$$g_{LA} + g_{NA} + g_{KA} + g_{MA} = 0 \quad (4)$$

The system comprised of (1), (2) and (3) subject to the set of restrictions (4) constituted the general joint cost model used in estimation.

---

[99]Symmetry is already imposed.

4.3 Testing the Structure of Technology

Various alternative hypotheses of the structure of the production technology were tested. Likelihood ratio tests, which have been extensively used to test the validity of various parameter restrictions, were employed in the following manner.

First, the general nonhomothetic joint cost model represented by (1), (2), (3), and (4) was estimated which yielded the maximum of the likelihood function L_0 upon convergence.[100] Second, the maximum of the likelihood function L_1 was estimated when additional alternative restrictions specified in section 3.2 were placed on the model. Third, the logarithm of the likelihood ratio was calculated by the formula $\ln (L_1 / L_0)$. Theil (1971) has shown that $-2(\ln L_1 - \ln L_0)$ is asymptotically distributed as Chi-squared. Hypothesis testing was conducted by comparing minus twice of the logarithm of the likelihood ratio to a critical value of X_r^2 where the degrees of freedom, r, is equal to the number of additional independent restrictions.[101]

---

[100] This specification converged after 35 iterations.

[101] Theil (1971), 396 - 397.

The null hypothesis or, in other words, the validity of the additional restrictions was rejected if minus twice the log of the likelihood ratio was greater than the critical value of X_r^2.

Tests were conducted on various null hypotheses such as global homotheticity, homogeneity, homogeneity of degree one and log-linearity. Since these restrictions are global tests, the translog cost function was treated as an exact function in this section. The results of the hypothesis testing are presented in Table 4 and Table 5.

Homotheticity implies input prices are separable from the levels of output and the cost minimization expansion path is a ray from the origin. Total weak homotheticity could not be tested within the translog framework. The null hypothesis of a strongly homothetic production technology was soundly rejected. A similar result was reported in Lopez (1980) for the single output case.

The test for input price and output separability is a test for the existence of an output aggregate.[102] Although the weak test could not be conducted, the rejection of strong homotheticity does question the appropriateness of the use of a single output specification. The acceptance of strong homotheticity is a sufficient condition for the

---

[102]Fuss and Waverman (1978), p. 21.

Table 4: Tests of the Production Structure

Hypothesis	Restrictions	Test Statistic $-2 \ln (L_1/L_0)$	D. F. r	Critical Value	Decision
General nonhomothetic	regularity conditions	—	—	—	
Homogeneous	$\sum_{\gamma} g_{iy} = 0, \sum_{\gamma} q_{yr} = 0$	63.22	5	15.09	reject H_0
Homogeneous of degree one	$\sum_{\gamma} g_{iy} = 0, \sum_{\gamma} q_{yr} = 0$ $\sum_{\gamma} q_y = 1$	78.7	6	16.81	reject H_0
Strongly homothetic	$g_{iy} = 0$	65.64	6	16.81	reject H_0
Strongly homothetic and homogeneous	$g_{iy} = 0$ $\sum_{\gamma} q_{yr} = 0$	66.36	8	20.09	reject H_0

Table 5: Tests of the Production Structure continued

Hypothesis	Restrictions	Test Statistic $-2 \ln (L_1/L_0)$	D. F. r	Critical Value	Decision
Homogeneous of degree one	$g_{iy} = 0$ $\sum_{Y} q_{yr} = 0, \sum_{Y} q_y = 1$	82.06	9	21.66	reject H_0
Strongly homothetic, strongly homogeneous	$g_{iy} = q_{yr} = 0$	143.96	9	21.66	reject H_0
Homogeneous of degree one	$g_{iy} = 0, \quad q_{yr} = 0$ $\sum_{Y} q_y = 1$	159.34	10	23.21	reject H_0
Homogeneous Cobb-Douglas	$g_{iy} = q_{yr} = b_{ij} = 0$	228.88	15	30.58	reject H_0
Cobb-Douglas homogeneous of degree one	$g_{iy} = q_{yr} = b_{ij} = 0$ $\sum_{Y} q_y = 1$	245.28	16	32.00	reject H_0

existence of an output aggregate. However, the rejection of strong homotheticity is neither necessary nor sufficient for the nonexistence of an output aggregate.[103] Rejection of strong homotheticity does suggest to view the single output analysis with caution.

Given the fact global strong homotheticity was rejected, this implies global homogeneity, linear homogeneity and log-linearity which are nested within strong homotheticity would also be rejected. The empirical tests led to the rejection of the null hypotheses of homogeneity nested within strong homotheticity in both tests. In addition, the nonnested homogeneous hypothesis was also rejected.

The null hypothesis of homogeneity of degree one was convincingly rejected in all four tests. The implication of these findings indicate the imposition of these restrictive specifications of linear homogeneity, homogeneity or strong homotheticity would bias any empirical findings for Canadian agriculture.

The log-linearity hypothesis is equivalent to testing

---

[103]This is due to the fact that the rejection of weak homotheticity is necessary and sufficient for the nonexistence of an consistent output aggregator.

for the Cobb-Douglas form of the cost function.[104] Log-linearity was strongly rejected which confirms the findings of Islam and Veeman (1980) who tested for this form in the single output case. Furthermore, the null hypothesis of the linear homogeneous Cobb-Douglas form was also rejected.

As a result of the empirical testing for global structure, the general, nonhomothetic, joint translog multiple input, multiple output cost function was maintained in all subsequent analysis and calculations. Accordingly, the imposition of homogeneity or linear homogeneity on the Canadian agriculture cost function would clearly be inappropriate.

4.4 Estimation Results and Summary Statistics

The parameter estimates of the full nonhomothetic joint multiple input multiple output cost model for Canadian agriculture are reported in Table 6 along with their associated standard errors and asymptotic t

---

[104] This is also a test for the Cobb-Douglas form of the production function since the Cobb-Douglas is a self dual.

ratios.[105] Twenty - eight of the thirty - four parameter estimates are statistically significant at the one percent level of confidence. Furthermore, three additional parameters are statistically significant at the five percent level.

Goodness of fit statistics are presented in Table 7 with R^2 values reported for each equation. An examination of this table indicates the model performed very well.

All the regional dummy variables are statistically significant. The negative coefficient for the dummy variable in the land and capital equation implies Western Canada has a relatively land and capital intensive production technology while the positive coefficient in the labor equation indicates a relatively labor intensive technology for Eastern Canada. The latter result is contrary to Lopez and Tung (1982) who found Western Canada to have a more labor intensive technology. This difference may be a result of either differences in the multiple output versus single output case or differences resulting from the functional form of the cost

---

[105]It is common to report the parameter estimates of the rejected models to show the extent of the bias in the parameter estimates if one assumes an inappropriate structure. Since nine alternative models were tested, in addition to the nonhomothetic joint model, it was decided not to report the rejected versions due to space limitations.

Table 6: Parameter Estimates for the Translog Cost Function

Coefficients	Estimated Value	Standard Error	Asymptotic t-Ratio
a_0	7.64140	.02595	294.432*
a_L	.41360	.00610	67.719*
a_N	.09308	.00375	24.807*
a_K	.23001	.00306	75.093*
a_M	.26331	.00995	26.459*
b_{LN}	-.06769	.00755	- 8.961*
b_{LK}	-.04268	.01108	- 3.852*
b_{LM}	-.08122	.01417	- 5.729*
b_{LL}	.19161	.01340	14.297*
b_{NN}	.04990	.00991	5.032*
b_{NK}	-.01965	.00677	- 2.902**
b_{NM}	.03744	.01128	3.319*
b_{KM}	.04128	.01464	2.819**
b_{KK}	.02106	.01422	1.480
b_{MM}	.00249	.02596	.095
q_F	.56129	.01812	32.667*
q_A	.55055	.01691	32.553*
g_{LF}	-.13868	.01149	-12.062*
g_{NF}	-.09535	.00635	-15.005*
g_{KF}	.00091	.01112	.081
g_{MF}	.23312	.01443	16.153*
g_{LA}	-.14271	.01443	- 9.885*
g_{NA}	-.06072	.00837	- 7.251*
g_{KA}	.04323	.01347	3.209**
g_{MA}	.16020	.18535	8.643*
q_{FA}	-.24027	.03909	- 6.146*
q_{FF}	.58468	.03381	17.290*
q_{AA}	.50687	.06176	8.207*
d_C	-.18079	.03802	- 4.754*
d_L	-.29900	.00886	-33.745*
d_N	.05220	.00550	9.482*
d_K	-.03438	.00412	- 8.334*
d_F	-.20897	.02538	- 8.232*
d_A	.47850	.02493	19.189*

*significant at the 1% level
**significant at the 5% level

Table 7: Goodness of Fit Statistics

Equation	R^2	SEE	DW
Cost function	.9449	.11659	.9956
Land cost share	.9720	.02560	1.1620
Labor cost share	.7736	.01600	1.1017
Capital cost share	.6718	.00935	.8890
Field revenue share	.8138	.07688	.9146
Animal revenue share	.9374	.07327	1.2308

function.[106]

The results from the two revenue share equations reflects relative specialization of field crops in the West and of livestock and animal products in the East. The results are not surprising given the geographical nature of Canada and the location of markets. The negative coefficient of the regional dummy in the cost function, although relatively small but statistically significant, indicates Eastern Canada has a lower cost curve compared to Western Canada. A summary of the interpretation of the regional dummy variables is presented in Table 8.

Table 9 presents the estimates of the elasticities of substitution for Canadian agriculture. Treating the translog cost function as a second order approximation requires the calculation of the elasticities to be conducted at the point of approximation. The elasticities of substitution formula is evaluated at the mean of the data.

The aggregate Canadian results indicate that all pairs of inputs are substitutes with the exception of land

---

[106] One should be careful in comparing the signs of the dummy variables in this study with those of Lopez and Tung (1982) since the former enters 0 for the West and 1 for the East while the latter enters 1 for the East and 0 for the East.

Table 8: Interpretation of Regional Dummy Variables

Equation	Western Canada	Eastern Canada
Land	Intensive	
Labor		Intensive
Capital	Intensive	
Field Crops	Specialization	
Livestock and Animal Products		Specialization
Costs	Higher	Lower

Table 9: Estimated Elasticities of Substitution for Canadian Agriculture[a]

	Land	Labor	Capital	Materials
Land	-0.28515	-0.71645	0.40793	0.28310
Labor		-3.91710	0.19475	1.97680
Capital			-3.26230	1.58890
Materials				-1.99070

[a]Estimated at the mean of the aggregate data based on the translog cost function.

and labor which reveals a complementary relationship. The complementary relationship between land and labor confirms the findings of Lopez and Tung (1982). Labor-materials and capital-materials exhibits the largest degree of substitutablity both having values of $E_{ij} > 1$. The remaining three E_{ij} values are positive but less than one.

Calculations of the elasticities of substitution were also conducted for each region. The substitution formula was evaluated at the mean of the regional data.

The elasticities of substitution estimates for Western Canada are reported in Table 10. All pairs of inputs are substitutes excluding land-labor and labor-capital.[107] Similar to the findings of the aggregate data, labor-materials and capital-materials show the largest substitution possibilities.

Table 11 shows the elasticities of substitution values for Eastern Canada. The Eastern Canada results are similiar to those found in the Canadian aggregate.

Due to the variability in the $AUES_{ij}$, upon examination of Tables 9, 10 and 11, one can clearly rule out the Cobb-Douglas technology where $AUES_{ij} = 1$, the Leontief technology where $AUES_{ij} = 0$ and the CES

---

[107] Labor-capital elasticity of substitution is statistically insignificant.

Table 10: Estimated Elasticities of Substitution for Western Canada[a]

	Land	Labor	Capital	Materials
Land	-0.29602	-0.91734	0.54160	0.29196
Labor		-3.84840	-0.04071	2.61030
Capital			-3.05800	1.67290
Materials				-2.60510

[a]Estimated at the mean of the Western data based on the translog cost function.

Table 11: Estimated Elasticities of Substitution for Eastern Canada[a]

	Land	Labor	Capital	Materials
Land	-0.04715	-0.74142	0.19285	0.21314
Labor		-3.50170	0.32663	1.65760
Capital			-3.48810	1.5328
Materials				-1.55520

[a]Estimated at the mean of the Eastern data based on the translog cost function.

specification where $AUES_{ij}$ are equal.

The own price and cross price elasticities of demand for the Canadian aggregate are shown in Table 12. All four inputs have the correct, a priori, sign for their own price elasticity of demand. The low value of E_{ii} for land indicates an inelastic demand function for this input. The low own price elasticity of demand for land is consistent with Binswager (1973) and Islam and Veeman (1980) but is different than the results of Lopez and Tung (1982). Materials and capital have the least relatively inelastic demand functions out of the four inputs. Only land-labor and labor-land show negative cross price elasticities.

The partial cross price and own price elasticities of factor demands were also calculated for Western and Eastern Canada. These results are reported in Table 13 and Table 14, respectively. In general, there appears to be no marked difference between the elasticity values for Canada as a whole and the regional values with the exception of capital-labor and labor-capital. In Western Canadian these values are negative indicating a complementary relationship while Eastern Canada and the Canadian aggregate identify these inputs as substitutes.

Evaluating the signs of the Allen-Uzawa partial elasticity of substitution and the cross partial

Table 12: Estimated Price Elasticities for Canadian Agriculture[a]

	Land	Labor	Capital	Materials
Land	-.09734	-.08277	.08616	.09395
Labor	-.24459	-.45255	.04113	.65601
Capital	.13926	.02249	-.68905	.52730
Materials	.09664	.22838	.33561	-.66063

[a]Estimated at the mean of the aggregate data based on the translog cost function.

Table 13: Estimated Price Elasticities for Western Canada[a]

	Land	Labor	Capital	Materials
Land	-.12354	-.07761	.12087	.08525
Labor	-.38284	-.32560	-.00908	.71750
Capital	.22603	-.00344	-.68244	.45984
Materials	.12184	.22085	.37335	-.71607

[a]Estimated at the mean of the Western data based on the translog cost function.

Table 14: Estimated Price Elasticities for Eastern Canada[a]

	Land	Labor	Capital	Materials
Land	-.01271	-.10858	.03842	.08287
Labor	-.19681	-.51282	.06508	.64456
Capital	.05119	.04783	-.69503	.59602
Materials	.05657	.24276	.30542	-.60474

[a]Estimated at the mean of the Eastern data based on the translog cost function.

elasticity of factor substitution allows one the to classify the inputs as complements and substitutes. Table 15 summarizes these findings for Canada, Western Canada and Eastern Canada.

The output elasticities of factor demand were calculated for Canada and the two regions. These results are presented in Table 16. Recall a negative sign indicates the factor input is an inferior good, a value between zero and positive one reveals the input is normal and a value greater than positive one identifies the input as a superior input.

Examining the field crop elasticities of factor demands reveals the following. Labor is an inferior input in all three calculations indicating the demand for labor shifts inward as the quantity of field crops increase. This is consistent with the observed relative decline in agricultural labor use. The calculation of the field crop elasticity of land reveals this factor of production is an inferior factor input in Eastern Canada and a normal input in Western Canada. It appears Eastern Canada field crop production is constrained the relative unavailability of land while the West is not. This supports the findings of Lopez and Tung (1982) who utilized a single aggregate output measure. It was found that capital used in field crops was a normal factor of production in both regions.

Table 15: Classification of Factor Inputs for Agriculture

	Land	Labor	Capital	Materials
Land				
Canada		complements	substitutes	substitutes
Western		complements	substitutes	substitutes
Eastern		complements	substitutes	substitutes
Labor				
Canada	complements		substitutes	substitutes
Western	complements		complements	substitutes
Eastern	complements		substitutes	substitutes
Capital				
Canada	substitutes	substitutes		substitutes
Western	substitutes	complements		substitutes
Eastern	substitutes	substitutes		substitutes
Materials				
Canada	substitutes	substitutes	substitutes	
Western	substitutes	substitutes	substitutes	
Eastern	substitutes	substitutes	substitutes	

Table 16: Output Elasticities of Factor Demand

	Canada	Western	Eastern
Field Crops			
Land	.09110	.32630	- .19234
Labor	- .32800	- .46840	- .31501
Capital	.50164	.66268	.34064
Materials	1.19980	1.50670	.93558
Livestock			
Land	.36930	.16518	.52374
Labor	.26175	- .21055	.65289
Capital	.99200	.70085	1.28450
Materials	1.27010	1.09000	1.47950

Finally, materials in Western Canada are superior, while materials in Eastern Canada are normal. This latter result is consistent with the observed increases in energy use, fertilizer, etc.

Summarizing the field crop elasticities of factor demands, the empirical results suggest increases in the production of field crops in the West is accompanied by relatively small increases in land and capital, a decrease in the usage of labor and relative larger increases in material usage. Contrarily, in the East, an increase in field crop production brings forth relative increases in capital and material use and relative decreases in land and labor.

The livestock elasticity of factor demands reflect a slightly different pattern of input responses as a result of livestock/animal products expansion, vis-a-vis, field crops. Land and capital are normal factors of production in the livestock industry for all three calculations with the exception of capital in Eastern Canada livestock where it appears to be a superior input. On the other hand, the livestock elasticity of demand for materials indicates that materials are superior in Eastern Canada, Western Canada and the aggregate livestock industry. Finally, labor used in the livestock industry is an inferior factor of production in Western Canada and normal in Eastern

Canada.

The Eastern livestock elasticities of factor inputs reflect land and labor as normal inputs in the livestock industry while capital and materials are both superior. It is interesting to note that the Western output elasticities of factor demands for both field crops and livestock have the same pattern of input responses.

The interregional differeces in input responses suggest the East and West have significantly different production technologies. This implies the model specification employed throughout this thesis may be inappropriate. Further research should estimate the production technology of Western and Eastern Canadian agriculture separately to determine if there exists any biases due to the specification in the present study.[108]

A summary classification of factor inputs according to their output elasticity of factor demand for field crops and livestock is presented in Table 17.

The marginal costs of production of field crops and livestock for Canada and the two regions is shown in Table 18. These marginal costs are difficult to interpretate due to the ambiguous units of output. However, the

---

[108]This presumes the analyst has a sufficiently large data set.

Table 17: Classifying Output Elasticities of Factor Demand

	Canada	Western	Eastern
Field Crops			
Land	normal	normal	inferior
Labor	inferior	inferior	inferior
Capital	normal	normal	normal
Materials	superior	superior	normal
Livestock			
Land	normal	normal	normal
Labor	normal	inferior	normal
Capital	normal	normal	superior
Materials	superior	superior	superior

estimated values of each marginal cost can be readily employed to calculate the marginal rates of product transformation. The marginal rates of transformation are also reported in Table 18.

A comparison of the marginal rate of transformation of field crops for livestock between Western and Eastern Canada leads to an interesting result. In Western Canada, the agriculture sector would have to give up approximately one unit of field crops in order to produce one extra unit of livestock. Contrarily, the agriculture sector in Eastern Canada would have to forgo approximately three units of field crops in order to produce an additional unit of livestock. This verifies the relative specialization of field crops in the West and livestock in the East as indicated by the signs of the coefficients for the dummy variables in the revenue share equations.

4.5 Economies of Scope and Economies of Scale

Recall jointness or cost complementarity is a local test for the existence of local economies of scope. If there exists local economies of scope, then there exists a cost saving from producing field crops and livestock together.

The nonjointness hypothesis was tested by scaling the

Table 18: Marginal Costs and Rates of Transformation

	Canada	Western	Eastern
MC_F	1232.90	1760.83	757.56
MC_A	2055.94	1616.35	2235.44
MRT_{FA}	1.6676	.9177	2.9508

data at the mean as indicated earlier. The empirical results indicate the nonjointness hypothesis could not be rejected. This suggests, in the aggregate, there is no local cost complementarity between the production of the two outputs and indicates the lack of local economies of scope. The statistical results for the test of nonjointness and local overall economies of scale are presented in Table 19.

Following Gillen and Oum (1983), the cross partial of the cost function with respect to the two outputs was evaluated at the scaled data point. The cost complementary test statistic evaluated at this point yielded CC_{yr} = .06 which indicates a clear lack of complementarity and hence no cost advantage to the production of field crops and livestock/animal products simultaneously.

One would expect the existence of local economies of scope at the micro level for mixed farming operations. However, in the aggregate, given the composition of the two outputs and their respective diversified inputs, there is no a priori reason to expect cost complementarity in the aggregate analysis.

Finally, a test of the null hypothesis of local overall constant returns to scale was conducted. Recall, the global tests conducted for linear homogeneity were all convincingly rejected. It may be the case, there exists

Table 19: Local Tests of the Production Structure

Hypothesis	Restrictions	Test Statistic $-2 \ln (L_1/L_0)$	D. F. r	Critical Value	Decision
Non-jointness	$q_{yr} = -q_y q_r$	1.8	1	6.63	cannot reject H_0
Local constant returns to scale	$q_y = 1$	16.96	1	6.63	reject H_0

linear homogeneity in the neighborhood of the point of approximation. The empirical results led to the rejection of local overall constant returns to scale.

Due to the rejection of the above hypothesis, two measures for the Canadian aggregate and one measure for each region was calculated to determine whether each of these regions was characterized by aggregate local economies or diseconomies of scale. At the industry level, this is equivalent to determining where the industry is characterized by increasing or decreasing costs.

At the scaled data mean, SE = $(\sum_{Y} q_y)^{-1}$ = .8994 which indicates in the Canadian aggregate, agriculture is characterized by overall diseconomies of scale or in otherwords Canadian agriculture is an increasing cost industry. This was checked by examining the inverse of the sum of output cost elasticities at the mean of the unscaled data. As a result, it was found that SE = .7784 for Canadian agriculture which verifies the above.

The inverse of the sum of the output cost elasticities was also calculated for each region evaluating the formula at the mean of each regional data subset. The scale elasticity for Western and Eastern Canada was estimated to be .8579 and .7125, respectively. Both Eastern and Western Canadian agriculture are

increasing cost industries. The literal interpretation of these two values indicated if output is expanded equiproportionately, say by 10 percent in each region, then cost would increase by 11.66 percent in the West and by 14.04 percent in the East.

This result coupled with the fact the dummy variable in the cost function indicates Eastern Canada agriculture has a lower cost curve seems at first glance contradictory. The seemingly conflicting evidence can be reconciled by the following. It appears that the West has a cost function which is characterized by a relatively flatter cost curve (if one thinks of the single output case) which lies to the the right and slightly upwards, vis-a-vis, the East cost function. This provides a sufficient explanation of the two facts.

In concluded this section, the Canadian agriculture technology is nonhomothetic, nonjoint and nonhomogeneous. Approximating statistics indicate that agriculture in Canada is an increasing cost industry.

Chapter V: SUMMARY and CONCLUSION

This chapter is divided into two sections. Section 5.1 summarizes the highlights of the major findings of this study. Section 5.2 indicates the limitations of this thesis and makes recommendations for future research.

5.1 Summary of Findings

This study has examined the characteristics of the production technology of Canadian agriculture using the dual cost function. Earlier empirical studies explored the characteristics of the agriculture production structure employing a single output specification. Recent developments in the theory of multiple output technologies has allowed this thesis to disaggregate agriculture output into field crops and livestock/animal products. Moreover, this study analyzed the production technology of Eastern and Western Canadian agriculture.

Using a multiple input, multiple output, nonhomothetic, joint, translog cost function, global tests for homotheticity, homogeneity, log-linearity and the more restrictive linear homogeneous structures were conducted.

The four alternative global linear homogeneous tests

were all soundly rejected which indicates the restrictive assumption of constant returns to scale is clearly inappropriate when describing Canadian agriculture. Furthermore, it was found the production structure of Canadian agriculture is clearly nonhomogeneous.

It was shown the existence of a consistent output aggregator which is necessary and sufficient for the use of a single aggregate output specification required separability between input prices and outputs. Although weak separability could not be tested, the statistical rejection of strong separability between input prices and outputs suggests to view the results of the single aggregate output specification with caution.

As a result of the hypothesis testing for global structure, the general nonhomothetic joint multiple input, multiple output translog cost function was maintained throughout the thesis.

The employment of dummy variables statistically showed Western Canadian agriculture was characterized by a relatively land and capital intensive technology specializing in field crops. Alternatively, agriculture in the East revealed a relative specialization in livestock and animal products, a relative labor intensive technology and a lower cost function.

The summary statistics of the Allen-Uzawa elasticities of substitution and the partial cross price and own price elasticities of demand revealed no surprising results. These results are not strictly comparable to those of the other studies cited in Chapter I due to the aggregation of inputs and disaggregation of outputs.

The interpretation of the output elasticities of factor demands led to the conclusion that the agriculture production technology utilized in the East is significantly different than in the West. Moreover, the Eastern Canadian agriculture technology in field crops and livestock/animal products is markedly dissimilar while the input response as a result of an increase in field crops and livestock in the West reflects a close degree of similarity in each industry.

The calculation of the marginal rate of product transformation showed the West must give up one unit of field crops to produce an extra unit of livestock/animal products. Contrarily, it was shown the East must forgo approximately three units of field crops to gain an additional unit of livestock/animal products.

The local test for cost complementarity indicated, in the aggregate, there exists no cost savings from the production of the two outputs simultaneously.

Furthermore, it was shown that agriculture in the East, West and Canada is an increasing cost industry.

It is concluded that the agriculture technology utilized in Eastern Canada is significantly different than in Western Canada.

5.2 Areas of Future Research

Although this study is the first to analyze Canadian agriculture using a multiproduct approach, there exists several areas which should be clarified in order to direct future research. There exists three major limitations of the present study: data limitations, functional specification, and technological change.

For any empirical research, the quality and quantity of the data is crucially important since the statistical results are only as good as the data. The present study employed data originating from Agriculture Canada and Statistics Canada. Although it was perhaps the best data available at the time, I am unsatisfied with its quality. In particular, the expenditures on land appears to be overstated while the cost of hired, operator and family labor appears to be understated. This, of course, may lead to statistical biases and therefore the results of this study should be viewed with caution.

The data used in this study represents the years 1961 to 1979 disaggregated by region. Since each region is heterogeneous in nature, the future availability of adequate provincial data should be used to make an interprovincial comparison. Moreover, if the data set is sufficiently large, future research should disaggregate inputs and outputs still further since the empirical results tend to be sensitive to the degree and method of aggregation.

The functional specification of the model employed is also an important issue as indicated in section 2.5 and 2.6. The present study employed the translog specification of the cost function. Consequently, the global measures of economies of scope and product-specific economies of scale could not be estimated. The choice of the translog functional form was based upon the limitations of the Shazam program used in estimation in this study and the lack of a sufficiently large data set. Future research should explore alternative specifications of the multiple output, multiple input Canadian agriculture cost function.

The functional form of the revenue share equations was based on the the translog specification and assumptions of perfect competition and cost minimization. If some subsectors of agriculture are not perfectly

competitive, then the model can be extended to incorporate economic behavior under a regulatory constraint and imperfect competition. It would be of interest to examine a disaggregated Canadian agriculture model reflecting the economic interaction between the competitive sector and the regulated sector.

The translog specification of the model was extended with the introduction of dummy variables into each equation. The statistical analysis indicate the regional agricultural technologies are substantially dissimiliar. It would be appropriate if the regional technologies were estimated separately. The necessary condition to implement the preceding is an extended regional data set.

In the empirical section of this thesis, it was assumed technological change was of the Hicks´ neutral type and was excluded from the estimating equations. This assumption was based on the results of Lopez (1980) and was employed due to an insufficiently large enough sample size. If a sufficiently large data set becomes available, researchers should definitely incorporate technological change into the full multiple product model. The effect of technological change should be empirical tested rather than assumed.

Realizing the limitations of this thesis, one has to remember this study is but one of the first to examine

agriculture from a multiproduct cost approach. The refinement of the multiple output, multiple input costing approach in conjunction with increases in the quality and quantity of data will allow future researchers to increase our understanding of the production technology of Canadian agriculture.

Bibliography

Allen, R. G. D. (1938), Mathematical Analysis for Economists London: MacMillan, 1938.

Appelbaum, E. (1979), "On the Choice of Functional Forms," International Economic Review 20 (June 1979), 449 - 458.

Arrow, K. J. and Intriligator, M. D. (1982), Handbook of Mathematical Economics, Vol. 2, Amsterdam: North Holland, 1982.

Bailey, E. E. and Friedlaender, A. F. (1982). "Market Structure and Multiproduct Industries," Journal of Economic Literature 20 (September 1982), 1024 - 1048.

Baumol, W. J., Panzar, J. C. and Willig, R. D. (1982), Contestable Markets and the Theory of Industry Structure. New York: Harcourt Brace Jovanovich, 1982.

Berndt, E. R. (1976), "Reconciling Alternative Estimates of the Elasticity of Substitution," Review of Economics and Statistics 58 (Febuary 1976), 59 -68.

Berndt, E. R. and Christensen, L. R. (1973), "The Internal Structure of Functional Relationships: Separability, Substitution and Aggregation," Review of Economic Studies 40 (July 1973), 403 - 410.

Berndt, E. R. and Christensen, L. R. (1973), "The Translog Function and the Substitution of Equipment, Structures and Labor in U.S. Manufacturing, 1929 - 68," Journal of Econometrics 1 (March 1973), 81 - 113.

Berndt, E. R. and Christensen, L. R. (1974), "Testing for the Existence of a Consistent Aggregate Index of Labor Inputs," American Economic Review 64 (June,1974) 391 -401.

Berndt, E. R. and Khaled, M. S. (1979), "Parametric Productivity Measurement and Choice among Flexible Functional Forms," Journal of Political Economy 87 (December 1979) 1220 - 1245.

Berndt, E. R., and Watkins, G. C., "Energy Prices and Productivity Trends in the Canadian Manufacturing Sector, 1957-1976," Ottawa: Economic Council of Canada, 1981.

Berndt, E. R., and Wood, D. O. (1975), "Technology, Prices, and the Derived Demand for Energy," Review of Economics and Statistics 57 (August 1975) 259 - 268.

Berndt, E. R., and Wood, D. O. (1979), "Engineering and Econometric Interpretations of Energy-Capital Complementary," American Economic Review 69 (June 1979) 342 -354.

Binswanger, H. P. (1974a), "A Cost Function Approach to the Measurement of Elasticities of Factor Demand and Elasticities of Substitution," American Journal of Agriculture Economics 56 (May 1974) 377 - 386.

Binswanger, H. P. (1974b), "The Measurement of Technical Change Biases with Many Factors of Production," American Economic Review 64 (December 1974) 964 - 976.

Blackorby, C., Primont, D., and Russell, R. R. (1977), "On Testing Separability Restrictions with Flexible Functional Forms," Journal of Econometrics 5 (March 1977) 195 - 209.

Blackorby, C., Primont, D., and Russell, R. R. (1978), Duality, Separability, and Functional Structure: Theory and Economic Applications. Amsterdam: North Holland, 1977.

Blackorby, C., and Russell, R. R. (1976), "Functional Structure and the Allen Partial Elasticities of Substitution: An Application of Duality Theory," Review of Economic Studies 43 (1976) 285 - 292.

Braeutigam, R. R, Daughety, A. F. and Turnquist, M. A. (1982), "The Estimation of a Hybrid Cost Function for a Railroad Firm," Review of Economic Studies 64 (August 1982), 394 - 404.

Brinkman, G. L. and Prentice, B. E., Multifactor Productivity in Canadian Agriculture: An Analysis of Methodology and Performance 1961 - 1980, Agriculture Canada, Regional Development Branch, 1983.

Brown, R. S, Caves, D. W. and Christensen, L. R. (1979), "Modelling the Structure of Cost and Production for Multiproduct Firms," Southern Economic Journal 46 (July 1979), 256 - 273.

Burgess, D. F. (1974a), "Production Theory and the Derived Demand for Imports," Journal of International Economics 4 (Febuary 1974), 103 - 117.

Burgess, D. F. (1974b), "A Cost Minimization Approach to Import Demand Equations," Review of Economics and Statistics 56 (May 1974), 225 - 234.

Burgess, D. F. (1975), "Duality Theory and Pitfalls in the Specification of Technologies," Journal of Econometrics

3 (May 1975), 105 - 121.

Caves, D. W., Christensen, L. R. and Diewert, W. E. (1982), "Multilateral Comparisons of Output, Input, and Productivity Using Superlative Index Numbers," Economic Journal 92 (March 1982), 73 - 86.

Caves, D. W., Christensen, L. R. and Swanson, J. A. (1981), "Productivity Growth, Scale Economies, and Capacity Utilization in U.S. Railroads, 1955 - 74," American Economic Review 71 (December 1981), 994 - 1002.

Caves, D. W., Christensen, L. R. and Tretheway, M. W. (1980), "Flexible Cost Functions for Multiproduct Firms," Review of Economics and Statistics 62 (August 1980), 477 - 481.

Chan, L. M. W. and Mountain, D. C. (1981), "The Measurement of Total Factor Productivity in Canadian Agriculture," University of Toronto Working Paper #8113.

Christensen, L. R. (1975), "Concepts and Measurements of Agricultural Productivity," American Journal of Agriculture Economics 57 (1975), 910 -915.

Christensen, L. R. and Greene, W. H. (1976), "Economies of Scale in U.S. Electric Power Generation," Journal of Political Economy 84 (1976), 655 - 676.

Christensen, L. R., Jorgenson, D. W. and Lau, L. J. (1973), "Transcendental Logarithmic Production Frontiers," Review of Economics and Statistics 55 (February 1973), 23 - 45.

Christensen, L. R. and Manser, M. E. (1977), "Estimating U.S. Consumer Preferences for Meat with a Flexible Utility Function," Journal of Econometrics 5 (January 1977), 37 - 53.

Corbo, V. and Meller, P. (1979), "The Translog Production Function: Some Evidence for Establishment Data," Journal of Econometrics 10 (1979), 193 - 199.

Darrough, M. and Heineke, J. (1978), "Multi-Output Translog Production Cost Function: The Case of Law Enforcement Agencies," in Heineke, J. (ed), Economic Models of Criminal Behavior, (Amsterdam: North Holland, 1978), 259 - 302.

Denny, M. (1974), "The Relationship between Functional Forms for the Production System," Canadian Journal of Economics 7 (February) 1974), 21 - 31.

Denny, M. and Fuss, M. (1977), "The Use of Approximation

Analysis to Test for Separability and Existence of Consistent Aggregates," American Economic Review 67 (June 1977), 404 - 418.

Denny, M. and May, D. (1977), "The Existence of a Real Value-Added Function in the Canadian Manufacturing Sector," Journal of Econometrics 5 (January 1977), 55 - 69.

Denny, M. and May, D. (1978a), "A Representation of Canadian Manufacturing Technology," Applied Economics 10 (December 1978), 305 - 317.

Denny, M. and May, D. (1978b), "Homotheticity and Real Value-Added in Canadian Manufacturing," in Fuss and McFadden (eds.), Production Economics: A Dual Approach to Theory and Applications, Vol. 2 (Amsterdam: North Holland, 1978),, 52 - 70.

Denny, M., Fuss, M., Everson, C., and Waverman, L. (1981) "Estimating the Effects of Diffusion of Technological Innovations in Telecommunications: The Production Structure of Bell Canada," Canadian Journal of Economics 14 (February 1981), 24 - 43.

Denny, M. and Pinto, C. (1978), "An Aggregate Model with Multi-Product Technologies," Fuss and McFadden (eds.), Production Economics: A Dual Approach to Theory and Applications, Vol. 2 (Amsterdam: North Holland, 1978), 247 - 268.

Dhrymes, P. J., Howrey, P. E., Hymans, S. H., Kmenta, J., Leamer, E. E., Quandt, R. E., Ramsey, J. B., Shapiro, H. T., and Zarnowitz, V. (1972), "Criteria for Evaluation of Econometric Models," Annals of Economic and Social Measurement 1 (July 1972), 291 - 324.

Diewert, W. E. (1971), "An Application of the Shepard Duality Theory: A Generalized Leontief Production Function," Journal of Political Economy 79 (May/June 1971), 759 - 774.

Diewert, W. E. (1973), "Functional Forms for Profit and Transformation Functions," Journal of Economic Theory 6 (June 1973), 284 - 316.

Diewert (1974a), "Functional Forms for Revenue and Factor Requirement Functions," International Economic Review 15 (1974), 119 - 130.

Diewert (1974b), "Applications of Duality Theory," in Intriligator and Kendick (eds.), Frontiers of Quantitative Economics, Vol. 2. (Amsterdam: North Holland, 1974), 106 - 171.

Diewert (1976), "Exact and Superlative Index Numbers," Journal of Econometrics 4 (May 1976), 115 - 146.

Diewert (1978), "Superlative Index Numbers and Consistency in Aggregation," Econometrica 46 (1978), 883 - 900.

Diewert (1982), "Duality Approaches to Microeconomic Theory," in Arrow and Intriligator (eds.), Handbook of Mathematical Economics, Vol. 2 (Amsterdam: North Holland, 1982), 535 - 600.

Freeman, R. B. and Medoff, J. B. (1979), "Substitution between Production Labor and Other Inputs in Unionized and Nonunionized Manufacturing," Review of Economics and Statistics (1979) .

Fuss, M. (1977), "The Demand for Energy in Canadian Manufacturing: An Example of the Estimation of Production Structures with Many Inputs," Journal of Econometrics 5 (January 1977), 89 - 116.

Fuss, M. (1978), "Factor Substitution in Electricity Generation: A Test of the Putty-Clay Hypothesis," in Fuss and McFadden (eds.), Production Economics: A Dual Approach to Theory and Applications, Vol. 2 (Amsterdam: North Holland, 1978),, 187 - 214.

Fuss, M. and McFadden, D. eds. (1978), Production Economics: A Dual Approach to Theory and Applications Vol. 1 and 2, Amsterdam: North-Holland, 1978.

Fuss, M., McFadden, D. and Mundlak, Y. (1978), "A Survey of Functional Forms in the Economic Analysis of Production," in Fuss and McFadden (eds.), Production Economics: A Dual Approach to Theory and Applications, Vol. 1 (Amsterdam: North Holland, 1978),, 217 - 268.

Fuss, M. and Waverman, L. (1978), "Multi-Product Multi-Input Cost Functions for a Regulated Utility: The Case of Telecommunications in Canada," Institute for Policy Analysis Working Paper #7810.

Gillen, D and Oum, T. (1983), "The Cost Structure of the Canadian Airline Industry and Refulatory Intervention: A Multi-Product Approach," Presented at the meetings of the Canadian Economics Association, Vancouver, 1983.

Griffin, J. M. (1979), "Statistical Cost Analysis Revisited," Quarterly Journal of Economics 93 (February 1979), 107 - 129.

Griffin, J. M. and Gregory, P. R. (1976), "An Intercountry Translog Model of Energy Substitution Responses," American Economic Review 66 (December 1976), 845 - 858.

Guilkey, D. K. and Lovell, C. A. K. (1980), "On the Flexibility of the Translog Approximation," International Economic Review 21 (February 1980), 137 - 147.

Hall, R. (1973), "The Specification of Technology with Several Kinds of Outputs," Journal of Political Economy 81 (1973), 878 - 892.

Halvorsen, R. (1977), "Energy Substitution in U.S. Manufacturing," Review of Economics and Statistics 59 (November 1977), 381 - 388.

Hamermesh, D. S. and Grant, J. (1979), "Econometric Studies of Labor - Labor Substitution and Their Implications for Policy," Journal of Human Resources 65 (Fall 1979), 518 - 542.

Hanoch, G. (1975), "Production and Demand Models with Direct or Indirect Implicit Additivity," .R. 43 (May 1975), 395 - 419.

Hellinger, F. J. (1975), "Specification of a Hospital Production Function," Applied Economics 7 (September 1975), 149 - 160.

Houthakker, H. S. (1965), "Self - Dual Preferences," .R. 33 (October 1965), 797 - 801.

Humphrey, D. B. and Moroney, J. R. (1975), "Substitution among Capital, Labor, and Natural Resource Products in American Manufacturing," Journal of Political Economy 83 (February 1975), 57 - 82.

Intriligator, M. D. (1978), Econometric Models, Techniques, and Application Englewood Cliffs, N.J.: Prentice-Hall, 1978.

Intriligator, M. D. and Kendrick, D. A.,eds. (1974) Frontiers of Quantitative Economics Vol. 2 Amsterdam: North-Holland, 1974.

Islam, T. S. (1982), "Input Substitution and Productivity Change in Canadian Agriculture," unpublished Ph. D. dissertation, University of Alberta, Edmonton, Alberta.

Islam, T. S. and Veeman, T. S. (1980), "Changing Input Use and Productivity in Canadian Agriculture," Canadian Journal of Agricultural Economics (August 1980), 38 - 47.

Kako, T, (1978), "Decomposition Analysis of Derived Demand for Factor Inputs: The Case of Rice Production in

Japan," American Journal of Agriculture Economics 60 (1978), 628 - 635.

Kiss, F., Karabadjian, S., and Lefebvre, B.J., (1981), "Economies of Scale and Scope in Bell Canada: Some Econometric Evidence," Presented at the conference "Telecommunications in Canada: Economic Analysis of the Industry" Montreal (March, 1981).

Lau, L. J. (1974), "Applications of Duality Theory: Comments," in Intriligator and Kendick (eds.), Frontiers of Quantitative Economics, Vol. 2. (Amsterdam: North Holland, 1974),, 176 - 199.

Lau, L. J. (1978), "Applications of Profit Functions," in Fuss and McFadden (eds.), Production Economics: A Dual Approach to Theory and Applications, Vol. 1 (Amsterdam: North Holland, 1978),, 133 - 216.

Lau, L. J. and Tamura, S. (1972), "Economies of Scale, Technical Progress, and the Nonhomothetic Leontieff Production Function: An Application to the Japanese Petrochemical Processing Industry," Journal of Political Economy 80 (December 1972), 1167 - 1187.

Lopez, R. E. (1980), "The Structure of Production and the Derived Demand for Inputs in Canadian Agriculture," American Journal of Agriculture Economics 62 (May 1980), 38 - 45.

Lopez, R. E. (1982), "Analysis of a Small Open Economy: The Case of Energy Prices in Canada," American Journal of Agriculture Economics (August 1982), 510 - 519.

Lopez, R. E. and MacMillan, J. (1979), "Econometric Analysis of the Canadian Regional Hired Farm Labor Market," Agriculture Canada Working Paper September 1979.

Lopez, R. E. . and Tung, F. (1982), "Energy and Non-Energy Input Substitution Possibilities and Output Scale Effects in Canadian Agriculture," Canadian Journal of Agricultural Economics (July 1982), 115 - 132.

Magnus, J. R. (1979), "Substitution between Energy and Non-Energy Inputs in the Netherlands, 1950 - 1974," International Economic Review 20 (June 1979), 465 - 484.

May, J. D. and Denny, M. (1979), "Post War Productivity in Canadian Manufacturing," Canadian Journal of Economics 12 (February 1979), 29 - 41.

McFadden, D. (1963), "Further Results on C.E.S. Production Functions," Review of Economic Studies 30 (1963), 73 -

83.

McFadden, D. (1978), "Cost, Revenue, and Profit Functions," in Fuss and McFadden (eds.), Production Economics: A Dual Approach to Theory and Applications, Vol. 1 (Amsterdam: North Holland, 1978),, 3 - 110.

Moroney, J. R. and Toevs, A. (1977), "Factor costs and Factor Use: An Analysis of Labor, Capital, and Natural Resource Inputs," Southern Economic Journal (October 1977), .

Mundlak, Y. (1968), "Elasticities of Substitution and the Theory of Derived Demands," Review of Economic Studies 35 (1968), 225 - 235.

Nadiri, M. I. (1982), "Producers Theory," in Arrow and Intriligator (eds.), Handbook of Mathematical Economics, Vol. 2 (Amsterdam: North Holland, 1982),, 431 - 490.

Ohta, M. (1975), "Production Technologies of the U. S. Boiler and Turbogenerator Industries and Hedonic Price Indexes for their Products: A Cost Function Approach," Journal of Political Economy 83 (February 1975), 1 - 26.

Panzar, J. C. and Willig, R. D. (1977), " Economies of Scale in Multi-Output Production," Quarterly Journal of Economics 91 (August 1977), 481 - 493.

Panzar, J. C. and Willig, R. D. (1981), "Economies of Scope," American Economic Review 71 (May 1981), 268 - 272.

Ray, S. C. (1982), "A Translog Cost Function Analysis of U.S. Agriculture, 1939 - 77," American Journal of Agriculture Economics (August 1982), 491 - 498.

Russell, R. R. and Wilkinson, M. (1979). Microeconomics: A Synthesis of Modern and Neoclassical Theory New York: John Wiley and Sons, 1979.

Silberberg, E. (1978). The Structure of Economics: A Mathematical Analysis New York: McGraw-Hill, 1978.

Simmons, P. and Weiserbs, D. (1979), "Translog Flexible Functional Forms and Associated Demand Systems," American Economic Review 69 (December 1979), 892 - 901.

Theil, H. (1971). Principles of Econometrics New York: John Wiley and Sons, 1971.

Theil, H. (1978). Introduction to Econometrics Englewood

Cliffs: Prentice-Hall, 1978.

Toda, Y. ( ), "Substitutability and Price Distortions in the Demand for Factors of Production: An Empirical Estimation," Applied Economics ( ), 203 - 217.

Toda, Y. (1976), "Estimation of a Cost Function when the Cost is not Minimum: The Case of Soviet Manufacturing Industries, 1958 - 1971," Review of Economics and Statistics 58 (August 1976), 259 - 268.

Uzawa, H. (1962), "Production Functions with Constant Elasticities of Substitution," Review of Economic Studies 29 (1962), 291 - 299.

Uzawa, H. (1964), "Duality Principles in the Theory of Cost and Production," International Economic Review 5 (1964), 216 - 220.

Varian, H. R. (1978), Microeconomic Analysis New York: Norton and Co., 1978.

Wales, T. J. (1977), "On the Flexibility of Flexible Functional Forms: An Empirical Approach," Journal of Econometrics 5 (March 1977), 183 - 193.

Wills, J. (1979), "Technical Change in the U. S. Primary Metals Industry," Journal of Econometrics 10 (1979), 85 - 98.

Woodland, A. D. (1976), "Modelling the Production Sector of an Economy: A Selective Survey and Analysis," University of British Columbia, Department of Economics Working Paper #76-21.

APPENDIX A

Given the output elasticity of total cost is defined by

$$E_{cy} = \frac{\delta \ln C(P,Q)}{\delta \ln Q_y} \tag{1}$$

expansion of the right handside of (1) yields

$$\frac{\delta \ln C(P,Q)}{\delta \ln Q_y} = \frac{\delta C(P,Q)}{C(P,Q)} \bigg/ \frac{\delta Q_y}{Q_y}$$

$$= \frac{\delta C(P,Q)}{\delta Q_y} \frac{Q_y}{C(P,Q)} \tag{2}$$

By definition the marginal cost of product y is

$$MC_y = \frac{\delta C(P,Q)}{\delta Q_y} \tag{3}$$

Now if profit maximizing occurs, then the following holds

$$MR_y = MC_y \tag{4}$$

Furthermore, perfection competition implies

$$P_y = MR_y \tag{5}$$

Substituting into (2) yields

$$\frac{\delta \ln C(P,Q)}{\delta \ln Q_y} = \frac{P_y \ Q_y}{C(P,Q)} \tag{6}$$

The numerator of (6) is total revenues recieved from the selling Q_y while the denominator of (6) is the total cost of producing output vector Q. Therefore, equation 6 is the revenue share of the y^{th} product.

APPENDIX B

The output elasticity of factor demand by definition is

$$E_{iy} = \frac{\delta \ln X_i}{\delta \ln Q_y}$$

$$= \frac{\delta X_i}{\delta Q_y} \frac{Q_y}{X_i} \qquad (1)$$

In order to find an expression for E_{iy}, given the translog cost function, one must find an expression for X_i and then the first derivative of X_i with respect to Q_y. By Shepard's lemma

$$X_i = \frac{\delta C(P,Q)}{\delta P_i} \qquad (2)$$

which is equal to

$$X_i = \frac{\delta \ln C}{\delta \ln P_i} \frac{C}{P_i} = \frac{\delta \ln C}{\delta \ln P_i} \frac{e^{\ln C}}{P_i} \qquad (3)$$

Partially differentiating (3) with respect to Q_y results in

$$\frac{\delta X_i}{\delta Q_y} = \frac{\delta \ln C}{\delta \ln P_i} \left[\frac{e^{\ln C}}{P_i} \quad \frac{\delta \ln C}{Q_y \delta \ln Q_y} \right] + \frac{\delta^2 \ln C}{Q_y \delta \ln P_i \delta \ln Q_y} \left[\frac{e^{\ln C}}{P_i} \right]$$

Recall, the cost shares and revenue shares

$$S_i = \frac{\delta \ln C}{\delta \ln P_i} \tag{5}$$

$$R_y = \frac{\delta \ln C}{\delta \ln Q_y} \tag{6}$$

respectively. Collecting common terms and substituting (5) and (6) into equation (4) then

$$\frac{\delta X_i}{\delta Q_y} = \frac{C}{P_i \; Q_y}\left[S_i \; R_y + \frac{\delta^2 \ln C}{\delta \ln P_i \, \delta \ln Q_y}\right] \tag{7}$$

In the translog model employed in this thesis

$$\frac{\delta^2 \ln C}{\delta \ln P_i \, \delta \ln Q_y} = g_{iy} \tag{8}$$

Substituting (8) into (7) implies

$$\frac{\delta X_i}{\delta Q_y} = \frac{C}{P_i \; Q_y}\left[S_i \; R_y + g_{iy}\right] \tag{9}$$

Substituting (9) into the definition of E_{iy} given in (1) yields

$$E_{iy} = \frac{C \; Q_y}{P_i \; X_i \; Q_y}\left[S_i \; R_y + g_{iy}\right] \tag{10}$$

Recalling the definition of the cost share indicates the first term on the right hand side of equation is the reciprocal of the cost share. Therefore (10) becomes

$$E_{iy} = \frac{1}{S_i}\left[S_i\ R_y + g_{iy}\right] \tag{11}$$

Finally, the above can be expressed as

$$E_{iy} = (g_{iy} / S_i) + R_y$$

which is exactly the expression given on page 71.